Matemáticas II
2º Bachillerato
Capítulo 10: Integrales

Respuestas a los ejercicios y problemas propuestos

www.apuntesmareaverde.org.es

Realizados por: AITANA, AITOR, AMALIA, ANDREA, CARMEN, CELIA S, FERNANDO, IRENE, ISMAEL F, LAURA, NATALIA, OLIVIA, ROSA, PABLO G, ESPERANZA, LIDIA, CELIA P, ISMAEL C, ENRIQUE, LUCÍA, NEREA, JULIA,

IES ATENEA, CIUDAD REAL

Revisor: Luis Carlos Vidal del Campo

Todas las imágenes han sido creadas con *software* libre (GeoGebra)

Actividades propuestas

1. Calcula las siguientes primitivas

a) $\int 4x^3 dx$ b) $\int 3x^2 dx$ c) $\int 5x^4 dx$ d) $\int (5x^4 - 4x^3 + 3x^2) dx$

a) $\int \mathbf{4x^3\, dx} = x^4 + C$

b) $\int \mathbf{3x^2\, dx} = \frac{3x^3}{3} + C = x^3 + C$

c) $\int \mathbf{5x^4 dx} = \frac{5x^5}{5} + C = x^5 + C$

d) $\int \mathbf{(5x^4 - 4x^3 + 3x^2) dx} = x^5 - x^4 + x^3 + C$

2. Dada $f(x) = (x^3 - 3x^2 + 2x + 1)$, calcula la primitiva F(x) de f(x) que verifica F(0) = 4

$F(x) = \int (\mathbf{x^3 - 3x^2 + 2x + 1})\mathbf{dx} = \frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} + x + C = \frac{x^4}{4} - x^3 + x^2 + x + C$

Como F(0) = 4 , F(0) = $\frac{0^4}{4} - 0^3 + 0^2 + 0 + C = 4$, luego C = 4 , de donde,

$F(x) = \frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} + x + 4$

3. Comprueba si $F(x) = (4x^3 + 2x^2 - x + 5)$ es una primitiva de $f(x) = (12x^2 + 4x + 3)$. En caso negativo, explica por qué.

La derivada de F(x) ha de ser igual a f(x)

Si derivamos F(x) obtenemos $F'(x) = 12x^2 + 4x - 1 \neq f(x)$
Por tanto, F(x) no es una primitiva de f(x)

4. Determina los valores de a, b, c y d para los que $(4a^3 + bx^2 + cx + d)$ es una primitiva de la función $f(x) = (4x^2 - 5x + 3)$

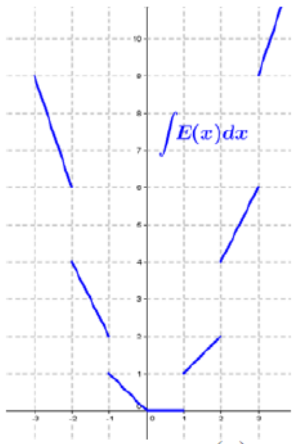
$F(x) = \int (4x^2 - 5x + 3)dx = 4\frac{x^3}{3} - 5\frac{x^2}{2} + 3x + C$ *, por tanto,*

$a = \frac{4}{3}$, $b = \frac{-5}{2}$, $c = 3$, $d = C$

5. Al resolver una primitiva, Javier y Ricardo han utilizado métodos diferentes y, como era de esperar, han obtenido expresiones distintas. Después de revisarlo muchas veces y no encontrar ningún error en los cálculos, le llevan el problema a la profesora para ver quien tiene bien el ejercicio. Para su sorpresa, la profesora les dice que ambos tienen bien el problema. ¿Cómo es posible?

Pueden diferir en constantes y estar los dos bien, además de las posibles simplificaciones.

6. Razona por qué la gráfica siguiente:



es una primitiva de la función "parte entera de x", $E(x)$, (salvo en los puntos de discontinuidad donde no es derivable):

$\int 1dx = x$ $\quad \int 2dx = 2x$ $\quad \int 3dx = 3x$ $\quad \int(-1)dx = -x$ $\quad \int(-2)dx = -2x$

7. Calcula las siguientes primitivas utilizando el cambio indicado:

a) $\int \frac{\sqrt{x}-\sqrt[3]{x}}{\sqrt[4]{x}}dx$ haciendo $x = t^{12}$.

$x = t^{12}$, $dx = 12t^{11}dt$, sustituyendo, obtenemos $\int \frac{\sqrt{t^{12}}-\sqrt[3]{t^{12}}}{\sqrt[4]{t^{12}}} \cdot 12t^{11}dt$, simplificando,

$$\int \frac{t^6-t^4}{t^3}12t^{11}dt = \int (t^6 - t^4)12t^8dt = 12\int(t^{14} - t^{12})dt = 12\left(\frac{t^{15}}{15} - \frac{t^{13}}{13}\right) + C$$

$x = t^{12}$, $t = \sqrt[12]{x}$, deshaciendo el cambio, $\int \frac{\sqrt{x}-\sqrt[3]{x}}{\sqrt[4]{x}} \cdot dx = 12\left(\frac{\left(\sqrt[12]{x}\right)^{15}}{15} - \frac{\left(\sqrt[12]{x}\right)^{13}}{13}\right) + C$

b) $\int \frac{dx}{e^x + e^{-x}}$ haciendo $e^x = t$.

$e^x = t$, $e^x dx = dt$, $dx = \frac{dt}{e^x} = \frac{dt}{t}$, $e^{-x} = \frac{1}{e^x} = \frac{1}{t}$ sustituyendo,

$$\int \frac{dx}{e^x+e^{-x}} = \int \frac{\frac{dt}{t}}{t+\frac{1}{t}} = \int \frac{\frac{dt}{t}}{\frac{t^2+1}{t}} = \int \frac{dt}{t^2+1} = arctg(t) + C = arctg(e^x) + C$$

c) $\int \frac{5x^4}{\sqrt{1+2x}}dx$ haciendo $1 + 2x = t^2$

$1 + 2x = t^2$, $2dx = 2tdt$, $dx = tdt$, $x = \frac{t^2-1}{2}$, $t = \sqrt{1+2x}$

$$\int \frac{5x^4}{\sqrt{1+2x}}dx = \int \frac{5\left(\frac{t^2-1}{2}\right)^4}{\sqrt{t^2}}tdt = 5\int \frac{(t^2-1)^4}{2^4}dt = \frac{5}{16}\int(t^8 - 4t^6 + 6t^4 - 4t^2 + 1)dt =$$

$$= \frac{5}{16}\left(\frac{t^9}{9} - 4\frac{t^7}{7} + 6\frac{t^5}{5} - 4\frac{t^3}{3} + t\right) + C =$$

$$= \frac{5}{16}\left(\frac{(\sqrt{1+2x})^9}{9} - 4\frac{(\sqrt{1+2x})^7}{7} + 6\frac{(\sqrt{1+2x})^5}{5} - 4\frac{(\sqrt{1+2x})^3}{3} + \sqrt{1+2x}\right) + C$$

d) $\int \frac{dx}{x+\sqrt{x^2-1}}$ haciendo $x+\sqrt{x^2-1}=t$

$x+\sqrt{x^2-1}=t$, $\sqrt{x^2-1}=t-x$, $\left(\sqrt{x^2-1}\right)^2 = (t-x)^2$,

$x^2-1 = t^2+x^2-2xt$, $2xt = t^2+1$, $x = \frac{t^2+1}{2t}$,

$dx = \frac{2t\cdot 2t-(t^2+1)\cdot 2}{4t^2}dt = \frac{(2t^2-2)}{4t^2}dt = \frac{t^2-1}{2t^2}dt$, de donde,

$$\int \frac{dx}{x+\sqrt{x^2-1}} = \int \frac{1}{t}\cdot\frac{t^2-1}{2t^2}dt = \frac{1}{2}\int\left(\frac{t^2}{t^3}-\frac{1}{t^3}\right)dt = \frac{1}{2}\int\left(\frac{1}{t}-t^{-3}\right)dt = \frac{1}{2}\left(ln|t| + \frac{t^{-2}}{2}\right)+C$$

Deshaciendo el cambio, $\int \frac{dx}{x+\sqrt{x^2-1}} = \frac{1}{2}\left(\ln\left|x+\sqrt{x^2-1}\right| + \frac{1}{2\left(x+\sqrt{x^2-1}\right)^2}\right)+C$

e) $\int\left(2\,\text{sen}^3 x + 3\,\text{sen}^2 x - \text{sen}\,x + 3\right)\cos x\,dx$ haciendo $\text{sen}\,x = t$

$\text{senx} = t$, $\text{cosxdx} = dt$, sustituyendo, nos queda,

$$\int (2t^3+3t^2-t+3)dt = 2\frac{t^4}{4} + 3\frac{t^3}{3} - \frac{t^2}{2} + 3t + C = \frac{\text{sen}^4x}{2} + \text{sen}^3x - \frac{\text{sen}^2x}{2} + 3\text{senx} + C$$

f) $\int \sqrt{1-x^2}dx =$ Haciendo x = sent ; t = arcsenx ; dx = costdt

$$\int \sqrt{1-sen^2t}\ cost\ dt = \int \sqrt{cos^2t}\cdot cost\ dt = \int cost\cdot cost\ dt = \int cos^2t\ dt = \int \frac{1+cos2t}{2}dt =$$

$$= \int \frac{1}{2}dt + \frac{1}{2}\int cos2t\ dt = \int \frac{1}{2}dt + \frac{1}{2}\cdot\frac{1}{2}\int 2cos2t\ dt = \frac{1}{2}t + \frac{1}{4}\cdot sent = \frac{1}{2}arcsenx + \frac{1}{4}x + C$$

8. Elige el cambio que simplifica las siguientes integrales:

a) $\int \frac{2x^3+1}{(x^4+2x)^3}dx$ $t = x^4+2x$, $dt = (4x^3+2)dx = 2(2x^3+1)dx$

b) $\int \frac{e^{\text{tg}x}}{\cos^2 x}dx$ $t = tgx$, $dt = \frac{1}{cos^2x}dx$

c) $\int \frac{\ln(\ln x)}{x\cdot\ln x}dx$ $t = ln(lnx)$, $dt = \frac{\frac{1}{x}}{lnx}dx = \frac{1}{xlnx}dx$

d) $\int 2x^3\sqrt{x^4-49}\cdot dx$

$t = x^4 - 49 \quad , \quad dt = 4x^3dx$

e) $\int \frac{x+1}{\sqrt[3]{x+1}+2}dx$

$t^3 = x + 1 \quad , \quad 3t^2dt = dx$

f) $\int \frac{x}{\sqrt{1-4x^2}}dx$

$t = 1 - 4x^2 \quad , \quad dt = -8xdx$

9. Determina si las siguientes integrales son inmediatas o no:

a) $\int\left(4x^3+3x^3-\frac{1}{x^2}+\sqrt{x}\right)dx$ SÍ.

b) $\int \frac{\ln x}{x}dx$ SÍ $\left(\int \frac{lnx}{x}dx = \int lnx\cdot\frac{1}{x}dx\right)$

c) $\int \text{sen}\,x\cos x\,dx$ SÍ

d) $\int \frac{e^{arcsenx}dx}{\sqrt{1-x^2}} = e^{arcsenx} + C$ SÍ

e) $\int \frac{\text{arctgx}}{1+x^2}\text{dx} = \frac{(\text{arctgx})^2}{2} + \text{C}$ SÍ

f) $\int \frac{\ln(x+1)}{x}dx$ NO

g) $\int tgx\cdot cosxdx = \int \frac{senx}{cosx}\cdot cosxdx = \int senxdx = -cosx$+C SÍ

h) $\int \frac{x^2-1}{\sqrt{1-x^2}}dx$ SÍ

i) $\int e^{x^2}dx$ NO

k) $\int \frac{x^4-2x^2+1}{x^2-1}dx$ SÍ $[x^4 - 2x^2 + 1 = (x^2-1)^2]$

j) $\int x^2\cdot e^{x^2}dx$ NO

10. Resuelve las siguientes integrales:

a) $\int\left(e^{3x}+e^{2x}+e^x\right)e^xdx$ $, t = e^x, dt = e^xdx, \int(t^3+t^2+t)dt = \frac{t^4}{4}+\frac{t^3}{3}+\frac{t^2}{2}+C = \frac{e^{4x}}{4}+\frac{e^{3x}}{3}+\frac{e^{2x}}{2}+C$

b) $\int(\ln x+2)\frac{dx}{x}$ $=\frac{(lnx+2)^2}{2}+C$

c) $\int \ln(\cos x)\operatorname{tg} x\,dx$, $t = \ln(cosx)$, $dt = -\frac{senx}{cosx}dx = -tgxdx =$

$= -\int tdt = -\frac{t^2}{2} + C = -\frac{(\ln(cosx))^2}{2} + C$

d) $\int \frac{x\,dx}{1+x^4}$ $=\frac{1}{2}\int \frac{2x}{1+(x^2)^2}dx = \frac{1}{2}arctg(x^2) + C$

i) $\int \frac{e^x dx}{1+e^{2x}}$ $=\int \frac{e^x}{1+(e^x)^2}dx = arctg(e^x) + C$

j) $\int x\cdot\cos e^{x^2}\cdot e^{x^2}dx$, $t = e^{x^2}$, $dt = 2xe^{x^2}dx$, $\frac{1}{2}\int costdt = \frac{1}{2}sent + C = \frac{1}{2}sen(e^{x^2}) + C$

11. Resuelve las siguientes integrales:

a) $\int (x^2+x+1)e^x dx$ $\begin{Bmatrix} u = x^2 + x + 1 & \rightarrow & du = (2x+1)dx \\ dv = e^x dx & \rightarrow & v = \int e^x dx = e^x \end{Bmatrix}$

$\int (x^2 + x + 1)e^x dx = (x^2 + x + 1)e^x - \int (2x+1)e^x dx =$

$\begin{Bmatrix} u = 2x+1 & \rightarrow & du = 2dx \\ dv = e^x dx & \rightarrow & v = \int e^x dx = e^x \end{Bmatrix}$ $= (x^2 + x + 1)e^x - [(2x+1)e^x - \int 2e^x dx] =$

$= (x^2 + x + 1)e^x - (2x+1)e^x + 2e^x + C = (x^2 - x + 2)e^x + C$

b) $\int \ln x\,dx$ $\begin{Bmatrix} u = lnx & \rightarrow & du = \frac{1}{x}dx \\ dv = dx & \rightarrow & v = \int dx = x \end{Bmatrix}$ $\int lnxdx = ln|x|x - \int x\frac{1}{x}dx = xln|x| - x + C$

c) $\int x\cos x\,dx$ $\begin{Bmatrix} u = x & \rightarrow & du = dx \\ dv = cosxdx & \rightarrow & v = \int cosxdx = senx \end{Bmatrix}$

$\int xcosxdx = xsenx - \int senxdx = xsenx + cosx + C$

Curiosidad – idea feliz: Resuelve la primitiva $\int \cos(\ln x)dx$.

Para ello, multiplica y divide el integrando por x: $\int \frac{\cos(\ln x)}{x}\cdot x\,dx = \left| \begin{matrix} u = x \rightarrow du = \ldots \\ dv = \frac{\cos(\ln x)}{x}dx \rightarrow v = \ldots \end{matrix} \right\rangle$

$I = \int \cos(lnx)\,dx = \int \frac{\cos(lnx)}{x}\cdot xdx =$ $\begin{Bmatrix} u = x & \rightarrow & du = dx \\ dv = \frac{\cos(lnx)}{x}dx & \rightarrow & v = sen(lnx) \end{Bmatrix}$

$= xsen(lnx) - \int sen(lnx)dx.$

$\int sen(lnx)dx = \int \frac{sen(lnx)}{x}\cdot xdx =$ $\begin{Bmatrix} u = x & \rightarrow & du = dx \\ dv = \frac{sen(lnx)}{x}dx & \rightarrow & v = -cos(lnx) \end{Bmatrix}$

$= -x\cos(\ln x) + \int \cos(\ln x)\,dx.$

$I = x\,sen(\ln x) + x\cos(\ln x) - I\,,\quad 2I = x\,sen(\ln x) + x\cos(\ln x)$

$\boldsymbol{I = \frac{1}{2}x(sen(\ln x) + \cos(\ln x)) + C}$

d) $\int \boldsymbol{arcsenx\,dx} =$ $\begin{bmatrix} u = arcsenx \to du = \frac{1}{\sqrt{1-x^2}}dx \\ dv = 1dx \to v = x \end{bmatrix}$ $=$

$arcsenx \cdot x - \int \frac{x}{\sqrt{1-x^2}}dx =$ $\left[t = 1 - x^2 \to dt = -2xdx \to \frac{dt}{-2} = xdx\right]$

$arcsenx \cdot x - \int \frac{\frac{dt}{-2}}{\sqrt{t}} = arcsenx \cdot x + \frac{1}{2}\int t^{\frac{-1}{2}}dt = arcsenx \cdot x + \frac{1}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}} =$

$= arcsenx \cdot x + t^{\frac{1}{2}} = arcsenx \cdot x + \sqrt{1-x^2} + C$

e) $\int \boldsymbol{senax \cdot e^{bx}dx} =$ $\begin{bmatrix} u_1 = sen(ax) \to du_1 = \text{a} \cdot \cos(ax)dx \\ dv_1 = e^{bx}dx \to v_1 = \frac{e^{bx}}{b} \end{bmatrix}$

$= \frac{e^{bx}sen(ax)}{b} - \int \frac{ae^{bx}\cos(ax)}{b}dx =$ $\begin{bmatrix} u_2 = \text{a} \cdot \cos(ax) \to du_2 = -a^2 sen(ax)dx \\ dv_1 = \frac{e^{bx}}{b} \to v_1 = \frac{e^{bx}}{b^2} \end{bmatrix}$

$= \frac{e^{bx}sen(ax)}{b} - \left(\frac{ae^{bx}\cos(ax)}{b^2} + \int \frac{a^2e^{bx}sen(ax)}{b^2}\right) =$

$= \frac{e^{bx}sen(ax)}{b} - \left(\frac{ae^{bx}\cos(ax)}{b^2} + \frac{a^2}{b^2}\int senax \cdot e^{bx}dx\right)$

$\frac{be^{bx}sen(ax) - ae^{bx}\cos(ax)}{b^2} - \frac{a^2}{b^2}\int senax \cdot e^{bx}dx$

Haciendo $I = \int senax \cdot e^{bx}dx$

$I = \frac{be^{bx}sen(ax) - ae^{bx}\cos(ax)}{b^2} - \frac{a^2}{b^2}I$

$I + \frac{a^2}{b^2}I = \frac{be^{bx}sen(ax) - ae^{bx}\cos(ax)}{b^2}\ ;\ \frac{a^2+b^2}{b^2}I = \frac{be^{bx}sen(ax) - ae^{bx}\cos(ax)}{b^2}$

De donde, $I = \frac{be^{bx}sen(ax) - ae^{bx}\cos(ax)}{a^2+b^2} + C$

12. Resuelve las siguientes primitivas.

a) $\int \frac{\mathbf{dx}}{\mathbf{x^2-4}}$ $\quad x^2 - 4 = 0;\ x_1 = 2, x_2 = -2$

$\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{A(x+2)+B(x-2)}{(x^2-4)} \to 1 = A(x+2) + B(x-2)$

$x = 2;\ 1 = 4A \to A = {}^1/_4$

$x = -2;\ 1 = -4B \to B = -{}^1/_4$

IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

$$\int \frac{dx}{x^2-4} = \int \frac{A}{(x-a)}dx + \int \frac{B}{(x-b)}dx = \int \frac{1/4}{x-2}dx + \int \frac{-1/4}{x+2}dx =$$

$$= \frac{1}{4}\int \frac{1}{x-2}dx - \frac{1}{4}\int \frac{1}{x+2}dx = \frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| = \frac{1}{4}(\ln|x-2| - \ln|x+2|) = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C$$

b) $\int \frac{dx}{(x+1)^2}$

$$\int \frac{dx}{(x+1)^2} = \int \frac{1}{(x+1)^2}dx = \int (x+1)^{-2}\,dx = \frac{(x+1)^{-1}}{-1} = \frac{-1}{x+1} + C$$

c) $\int \frac{x\,dx}{(x+1)^2}$ $\qquad (x+1)^2 = 0;\;\; x_1 = -1, x_2 = -1$

$$\frac{x}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} = \frac{A(x+1)+B}{(x+1)^2} \;\rightarrow\; x = A(x+1) + B$$

$$x = -1;\;\; B = -1$$

$$x = 0;\;\; A = 1$$

$$\int \frac{x\,dx}{(x+1)^2} = \int \frac{1}{x+1}dx + \int \frac{-1}{(x+1)^2}dx = \int \frac{1}{x+1}dx - \int (x+1)^{-2}\,dx =$$

$$= \ln|x+1| - \frac{(x+1)^{-1}}{-1} = \ln|x+1| + \frac{1}{x+1} + C$$

d) $\int \frac{x^3}{(x+1)^2}\mathbf{dx}$, efectuamos la división, obtenemos C(x) = x – 2 ; R(x) = 3x + 2, luego

$\int \frac{x^3}{(x+1)^2}dx = \int (x-2)dx + \int \frac{3x+2}{x^2+2x+1}dx$, como $x^2 + 2x + 1 = (x+1)^2$

$\int \frac{x^3}{(x+1)^2}dx = \int (x-2)dx + \frac{3}{2}\int \frac{2x}{x^2+2x+1}dx + 2\int \frac{1}{(x+1)^2}dx =$

$\frac{x^2}{2} - 2x + \frac{3}{2}\ln|x^2 + 2x + 1| - \frac{2}{x+1} + C$

e) $\int \frac{x^2+x+1}{x^3-4x^2+4x}dx = \int \frac{x^2+x+1}{(x^2-4x+4)(x)}dx = \int \frac{x^2+x+1}{(x)(x-2)^2}dx$

$\frac{x^2+x+1}{(x)(x-2)^2} = \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$

$x^2 + x + 1 = A(x-2)^2 + B(x)(x-2) + C(x)$

$x = 0;\;\; 1 = 4A \rightarrow A = \frac{1}{4}$

$x = 2;\;\; 7 = 2C \rightarrow C = \frac{7}{2}$

$x = 1;\;\; 3 = \frac{1}{4} - B + \frac{7}{2} \rightarrow \frac{-3}{4} = -B \rightarrow \frac{3}{4} = B$

$\int \frac{x^2+x+1}{(x)(x-2)^2}dx = \frac{1}{4}\int \frac{1}{x}dx + \frac{3}{4}\int \frac{1}{x-2}dx + \frac{7}{2}\int \frac{1}{(x-2)^2}dx \rightarrow \frac{1}{4}\ln|x| + \frac{3}{4}\ln|x-2| - \frac{7}{2}\left(\frac{1}{x-2}\right) + C$

f) $\int \frac{3x^2+1}{(2x-1)(3x^2+2)}dx$; $\frac{3x^2+1}{(2x-1)(3x^2+2)} = \frac{A}{2x-1} + \frac{Mx+N}{3x^2+2}$

$\rightarrow 3x^2 + 1 = A(3x^2 + 2) + (Mx + N)(2x - 1)$

$x = 0 \rightarrow 1 = 2A - N \rightarrow N = 2A - 1 \rightarrow 2\left(\frac{7}{11}\right) - 1 \qquad \rightarrow N = \frac{3}{11}$

$x = 1 \rightarrow 4 = A(5) + (M + N) \rightarrow 4 = 5A + M + N \rightarrow 4 = 5A + M + 2A - 1 \quad \rightarrow M = \frac{6}{11}$

$x = 2 \rightarrow 13 = 14A + (2M + N)3 \rightarrow 13 = 14A + 6M + 3N \qquad \rightarrow A = \frac{7}{11}$

$\int \frac{3x^2+1}{(2x-1)(3x^2+2)}dx = \frac{7}{11}\int \frac{1}{2x-1}dx + \int \frac{6x+3}{11(3x^2+2)}dx = \frac{7}{22}\int \frac{2}{2x-1}dx + \frac{1}{11}\int \frac{6x+3}{3x^2+2}dx =$

$= \frac{7}{22}\ln|2x - 1| + \frac{1}{11}\int \frac{6x+3}{3x^2+2}dx = \frac{7}{22}\ln|2x - 1| + \frac{1}{11}\int \frac{6x}{3x^2+2}dx + \frac{3}{11}\int \frac{\sqrt{3}}{(\sqrt{3}x)^2+2}dx =$

$= \frac{7}{22}\ln|2x - 1| + \frac{1}{11}\ln|3x^2 + 2| + \frac{3}{11}\cdot\frac{1}{\sqrt{2}}arctan\left(\frac{\sqrt{3}x}{\sqrt{2}}\right) + C$

g) $\int \frac{x^2-2}{x^3(x^2+1)}dx \rightarrow \frac{x^2-2}{x^3(x^2+1)} = \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{Dx+E}{x^2+1} \rightarrow$

$\rightarrow x^2 - 2 = A(x^2 + 1) + B\big(x(x^2 + 1)\big) + Cx^2(x^2 + 1) + (Dx + E)x^3 \rightarrow$

$x = 0;\ -2 = A$

$$\begin{cases} x = -1;\ -1 = -4 - 2B + 2C - (-D + E) \\ x = 1;\ -1 = -4 + 2B + 2C + D + E \\ x = 2;\ 2 = -10 + 10B + 20C + 8\,(2D + E) \\ x = -2;\ 2 = -10 - 10B + 20C - 8\,(-2D + E) \end{cases} \rightarrow F_1 + F_2 \rightarrow 5F_1 + F_3 \rightarrow -5F_1 + F_4$$

$$\rightarrow \begin{cases} 3 = -2B + 2C + D - E \\ 6 = 4C + 2D \\ 27 = 30C + 21D + 3E \\ -3 = 10C + 11D - 3E \end{cases} \rightarrow B = 0; C = 3; D = -3; E = 0 \rightarrow$$

$\int \frac{x^2-2}{x^3(x^2+1)}dx = \int \frac{-2}{x^3}dx + \int \frac{3}{x}dx + \int \frac{-3x}{x^2+1}dx = -2\int \frac{1}{x^3}dx + 3\int \frac{1}{x}dx - \frac{3}{2}\int \frac{2x}{x^2+1}dx =$

$= -2\frac{x^{-2}}{-2} + 3\ln|x| - \frac{3}{2}\ln|x^2 + 1| + C = \frac{1}{x^2} + 3\ln|x| - \frac{3}{2}\ln|x^2 + 1| + C$

h) $\int \frac{x^3+2x^2+5x+3}{x^2+1}dx = \qquad (x^3 + 2x^2 + 5x + 3): (x^2 + 1); \quad r(x) = 4x + 1; \quad c(x) = x + 2$

$= \int (x + 2 + \frac{4x+1}{x^2+1})dx = \int x\,dx + \int 2\,dx + \int \frac{4x}{x^2+1}dx + \int \frac{1}{x^2+1}dx =$

$= \frac{x^2}{2} + 2x + 2\,ln(x^2 + 1) + arctg(x) + C$

i) $\int \frac{x+1}{(x-1)(x+1)^2(x^2+1)}dx \rightarrow \frac{x+1}{(x-1)(x+1)^2(x^2+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)} + \frac{Dx+E}{(x^2+1)} \rightarrow$

$x + 1 = A(x + 1)^2(x^2 + 1) + B(x - 1)(x^2 + 1) + C(x - 1)(x + 1)(x^2 + 1) + (Dx + E)(x - 1)(x + 1)^2$

IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

$x=-1;\ \ 0=4B;\ \ \ B=0$

$x=1;\ \ \ 2=8A;\ \ \ A=4$

$$\begin{cases} x=0; \quad 1=4-C-E \\ x=-2; \quad -1=20-9C+6D-3E \\ x=2; \quad 3=180+15C+18D+9E \end{cases} \rightarrow C=-7; D=-9; E=10$$

$$\int \frac{x+1}{(x-1)(x+1)^2(x^2+1)}dx = \int \frac{4}{(x-1)}dx + \int \frac{-7}{(x+1)}dx + \int \frac{-9x+10}{(x^2+1)}dx =$$

$$= 4\int \frac{1}{(x-1)}dx - 7\int \frac{1}{(x+1)}dx + \int \frac{-9x}{(x^2+1)}dx + 10\int \frac{1}{(x^2+1)}dx =$$

$$= 4\ln|x-1| - 7\ln|x+1| - \frac{9}{2}\ln|x^2+1| + 10arctgx + C$$

13. Halla las siguientes primitivas:

a) $\int \mathbf{sen}\left(\frac{3}{2}x+1\right)\mathbf{dx}$; $\quad t=\frac{3}{2}x+1 \quad \frac{dt}{dx}=\frac{3}{2} \rightarrow dx=\frac{2\cdot dt}{3}$

- Ahora sustituimos en la integral y la resolvemos.

$$\int \operatorname{sen}(t)\frac{2\cdot dt}{3} \rightarrow \frac{2}{3}\cdot\int \operatorname{sen}(t)\,dt \rightarrow \frac{2}{3}\cdot(-\cos t) \rightarrow$$

- Sustituimos otra vez ***t*** por lo que teníamos y obtendremos el resultado.

$$\rightarrow -\frac{2\cdot cos\left(\frac{3}{2}x+1\right)}{3}+C$$

b) $\int \frac{\boldsymbol{sen(3x)}}{\sqrt[3]{\boldsymbol{cos(3x)}}}\boldsymbol{dx} = \int \frac{\text{sen}(3x)}{(\cos(3x))^{\frac{1}{3}}}dx$

t = cos (3x) → $\frac{dt}{dx}=-sin(3x)\cdot 3 \rightarrow dx=\frac{dt}{-sen(3x)\cdot 3}$

Ahora lo sustituimos y realizamos la integral

$$\int \frac{\cancel{sen(3x)}}{t^{\frac{1}{3}}}\cdot\frac{dt}{-\cancel{sen(3x)}\cdot 3} \rightarrow \int -\frac{1}{t^{\frac{1}{3}}}\cdot\frac{1}{3}\,dt \rightarrow \int -\frac{1}{3t^{\frac{1}{3}}}dt \rightarrow -\frac{1}{3}\int \frac{1}{t^{\frac{1}{3}}}dt$$

$$\frac{1}{3}\int t^{-\frac{1}{3}}\,dt \rightarrow -\frac{1}{3}\cdot\frac{t^{\frac{2}{3}}}{\frac{2}{3}} \rightarrow -\frac{1}{3}\cdot\frac{3\sqrt[3]{\cos(3x)^2}}{2} \rightarrow -\frac{\sqrt[3]{\cos(3x)^2}}{2}+C$$

c) $\int \frac{\boldsymbol{cotg(x)}}{\boldsymbol{sen^2(x)}}\boldsymbol{dx} = \int \frac{\frac{\cos(x)}{\text{sen}(x)}}{\text{sen}^2(x)}dx \rightarrow \int \frac{\cos(x)}{\text{sen}^3(x)}\,dx$

$$t=\text{sen}(x)\ ;\ \frac{dt}{dx}=\cos(x) \rightarrow dx=\frac{dt}{\cos(x)};$$

$$\int \frac{\cancel{\cos(x)}}{t^3}\cdot\frac{dt}{\cancel{\cos(x)}} \rightarrow \int \frac{1}{t^3}\,dt \rightarrow \int t^{-3}\,dt \rightarrow \frac{t^{-2}}{-2} \rightarrow -\frac{1}{2\text{sen}(x)^2}+C$$

$$\int \frac{cotg(x)}{sen^2(x)}dx = -\frac{1}{2\mathrm{sen}(x)^2} + \mathrm{C}$$

d) $\int \frac{\mathbf{sen\,2}x}{(\mathbf{cos\,2}x+\mathbf{1})^2}\,dx = -\frac{1}{2}\int \frac{(-2)\cdot \mathrm{sen}\,2x}{(\cos 2x+1)^2}dx = -\frac{1}{2}\cdot\frac{-1}{(\cos 2x+1)} + c = \frac{1}{2(\cos 2x+1)} + C$

$t = (\cos 2x + 1)$ $dt = (-2)\cdot \mathrm{sen}\,2x\,dx$

e) $\int (\tan x)^2\,dx = \int (1 + (\tan x)^2 - 1)\,dx = \int (1 + (\tan x)^2)\,dx - \int 1\,dx = \tan x - x + C$

f) $\int ((\tan x)^2 + x + 1)\,dx = \int (1 + (\tan x)^2 + x)\,dx = \int (1 + (\tan x)^2)\,dx + \int x\,dx =$

$= \tan x + \frac{x^2}{2} + C$

g) $\int \frac{\tan x}{cos^2 x}dx = \int \frac{1}{cos^2 x}(\tan x)dx = \frac{tan^2 x}{2} + C$

h) $\int \frac{dx}{1-sen^2 x}$ caso general $\begin{bmatrix} t = tg\frac{x}{2} & \cos x = \frac{1-t^2}{1+t^2} \\ dx = \frac{2\cdot dt}{1+t^2} & senx = \frac{2t}{1+t^2} \end{bmatrix}$

$$\int \frac{dx}{1-sen^2x} = \int \frac{\frac{2\cdot dt}{1+t^2}}{1-\frac{2t}{1+t^2}} = \int \frac{\frac{2}{1+t^2}}{\frac{1+t^2-2t}{1+t^2}}dt = \int \frac{2}{(t-1)^2}dt = \frac{2\cdot(t-1)^{-1}}{-1} + C = \frac{-2}{tg\frac{x}{2}-1} + C$$

i) $\int \frac{dx}{senx} = \int \frac{1}{senx}dx$ Caso: $\frac{1}{-senx} = -\frac{1}{senx}$ Impar en seno

$cosx = t\,;$ $senx = \sqrt{1-cos^2x} = \sqrt{1-t^2}$

$-senxdx = dt\,;\; dx = \frac{dt}{-senx} = -\frac{dt}{\sqrt{1-t^2}}$

$\int \frac{dx}{senx} = \int \frac{1}{senx}dx = \int \frac{1}{\sqrt{1-t^2}}\cdot\frac{-dt}{\sqrt{1-t^2}} = \int \frac{1}{t^2-1}dt$

$t^2 - 1 = 0 \to t = \pm 1 \quad \to \frac{1}{t^2-1} = \frac{A}{t-1} + \frac{B}{t+1}$

$1 = A(t+1) + B(t-1)$

$t = 1 \;\; ; \;\; 1 = A2 \;\; \to A = \frac{1}{2}$

$t = -1 \;\; ; \;\; 1 = -2B \;\; \to B = -\frac{1}{2}$

$\int \frac{1}{t^2-1}dt = \int \frac{\frac{1}{2}}{t-1}dt + \int \frac{-\frac{1}{2}}{t+1}dt \;\to\; \frac{1}{2}\ln|t-1| - \frac{1}{2}\ln|t+1| =$

$= \frac{1}{2}(\ln|cosx - 1| - \ln|cosx + 1|) + C = \frac{1}{2}\ln\frac{cosx-1}{cosx+1} + C$

j) $\int \mathbf{sen^2} \boldsymbol{x} \cos \boldsymbol{x}\, \boldsymbol{dx}$; $\quad \operatorname{sen} x = t\,,\ \cos x\, dx = dt\,;$

$$\int \operatorname{sen}^2 x \cos x\, dx = \int t^2 dt = \frac{t^3}{3} + C = \frac{\operatorname{sen}^3 x}{3} + C$$

k) $\int \mathbf{sen^2} \boldsymbol{x dx}$; $\quad \operatorname{sen}^2 x = \frac{1-\cos 2x}{2}$;

$$\int \operatorname{sen}^2 x dx = \int \frac{1-\cos 2x}{2} dx = \frac{1}{2}\int (1-\cos 2x) dx = \frac{1}{2}\int 1 dx - \frac{1}{2}\int \cos 2x dx =$$

$$\frac{1}{2}x - \frac{1}{2}\int \cos 2x dx = \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\int \cos 2x \cdot 2dx = \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x + C = \frac{x}{2} - \frac{\operatorname{sen} 2x}{4} + C$$

l) $\int \mathbf{sen^4} \boldsymbol{x dx}$; $\quad \operatorname{sen}^2 x = \frac{1-\cos 2x}{2}$; $\quad \cos^2 x = \frac{1+\cos 2x}{2}$

$$\int \operatorname{sen}^4 x dx = \int (\operatorname{sen}^2 x)^2 dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx = \int \frac{(1-\cos 2x)^{\ 2}}{2^2} dx = \int \frac{1-2\cos 2x + (\cos 2x)^2}{4} dx =$$

$$= \frac{1}{4}\int 1 dx - \frac{1}{4}\int 2\cos 2x\, dx + \frac{1}{4}\int \frac{1+\cos 4x}{2} dx = \frac{x}{4} - \frac{sen 2x}{4} + \frac{1}{4}\cdot\frac{1}{2}\int 1 dx + \frac{1}{4}\cdot\frac{1}{2}\int \cos 4x\, dx =$$

$$\frac{x - sen 2x}{4} + \frac{x}{8} + \frac{1}{8}\cdot\frac{1}{4}\int \cos 4x \cdot 4\, dx = \frac{3x}{8} - \frac{sen 2x}{4} + \frac{\operatorname{sen} 4x}{32} + C$$

m) $\int (\cos x)^4 dx = \int ((\cos x)^2)^2\, dx = \frac{1}{2}\int (1+\cos 2x)^2\, dx =$

$$= \frac{1}{2}\int 1 + 2\cos 2x + (\cos 2x)^2 dx = \frac{1}{2}\int 1dx + \frac{1}{2}\int 2\cos 2x\, dx + \frac{1}{4}\int 1 + \cos 4x =$$

$$= \frac{1}{2}\int 1dx + \frac{1}{2}\int 2\cos 2x\, dx + \frac{1}{4}\int 1dx + \frac{1}{8}\int 4\cos 4x\, dx =$$

$$= \frac{1}{2}x + \frac{1}{2}\operatorname{sen} 2x + \frac{1}{4}x + \frac{1}{8}\operatorname{sen} 4x + C = \frac{1}{8}\operatorname{sen} 4x + \frac{1}{2}\operatorname{sen} 2x + \frac{3}{4}x + C$$

n) $\int \cos(\ln x)\, dx$

u=$\cos(\ln x)$ $\qquad$ dv= dx

$du = -\operatorname{sen}(lnx)\frac{1}{x}dx \qquad v = x$

$$\int \cos(\ln x)\, dx = x\cdot\cos\ln x - \int \cancel{x}\left(-\operatorname{sen}(lnx)\frac{1}{\cancel{x}}\right) dx = x\cos(\ln x) + \int \operatorname{sen}(\ln x)\, dx$$

$$\int u\, dv = u\, v - \int v\, du$$

u= $\operatorname{sen}(\ln x)$ $\qquad$ dv= dx

$du = \cos(\ln x)\frac{1}{x}dx \qquad v = x$

$$x\cos(\ln x) + x\operatorname{sen}(\ln x) - \int \cancel{x}\cos(\ln x)\frac{1}{\cancel{x}}dx =$$

$$= x\cos(\ln x) + x\operatorname{sen}(\ln x) - \int \cos(\ln x) dx$$

$$\int \cos(\ln x)\,dx = x\cos(\ln x) + x\,\mathrm{sen}(\ln x) - \int \cos(\ln x)dx$$

$$I = x\cos(\ln x) + x\,\mathrm{sen}(\ln x) - I$$

$$2I = x\cos(\ln x) + x\,\mathrm{sen}(\ln x)\,; \qquad I = \frac{x\cos(\ln x) + x\,\mathrm{sen}(\ln x)}{2}$$

$$\int \cos(\ln x)\,dx = \frac{x\cos(\ln x) + x\,\mathrm{sen}(\ln x)}{2} + C$$

ñ) $\int \frac{(1+(\sin x)^2)}{\sin x \cos x} dx = \int \frac{1}{\mathrm{sen}\, x\cos x}dx + \int \frac{(\mathrm{sen}\, x)^{\not 2}}{\not{\mathrm{sen}\, x}\cos x}dx$

*Hacemos la primera integral:

$$\int \frac{1}{\mathrm{sen}\, x\cos x}dx$$

Sabemos que:

$$\left[\begin{array}{l} \tan\frac{x}{2} = t \to dx = \frac{2}{1+t^2}dt\,; \quad \mathrm{sen}\, x = \frac{2t}{1+t^2}\ ; \ \cos x = \frac{1-t^2}{1+t^2} \\ \qquad \mathrm{sen}\, x\cos x = \frac{2t}{1+t^2}\cdot\frac{1-t^2}{1+t^2} = \frac{2t-2t^3}{(1+t^2)^2} \end{array}\right.$$

$$\int \frac{1}{\mathrm{sen}\, x\cos x}dx = \int \frac{1}{\frac{2t-2t^3}{(1+t^2)^2}}\cdot\frac{2}{1+t^2}\,dt = \int \frac{(1+t^2)^2}{2t-2t^3}\cdot\frac{2}{1+t^2}dt = \int \frac{2(1+t^2)^2}{2t+2t^3-2t^3-2t^5}dt =$$

$$= \frac{\not 1}{\not 2}\cdot\not 2\int \frac{(1+t^2)^2}{t-t^5}dt = \int \frac{(1+t^2)^2}{t(1-t^4)}dt = \int \frac{(1+t^2)^{\not 2}}{t(1-t^2)\not{(1+t^2)}}dt = \int \frac{1+t^2}{t(1-t^2)}\mathrm{dt}$$

$\frac{1+t^2}{t(1-t^2)} = \frac{A}{t} + \frac{B}{1-t} + \frac{C}{1+t};\qquad 1+t^2 = A(1-t^2) + Bt(1+t) + Ct(1-t)$

*Calculamos A, B y C dando valores a t:

-Si t = 0; 1 = A$(1-0^2)$; $1 = A$

-Si t = 1; 2 = B·1·(1+1); 1 = B

-Si t=-1; 2 = 2C ; 1 = C

*Sustituimos los valores:

$$\int \left(\frac{1}{t} + \frac{1}{1-t} + \frac{1}{1+t}\right)dt = \ln|t| - \ln|1-t| + \ln|1+t|$$

*Sustituimos t por tan $\frac{x}{2}$:

$$\int \frac{1}{\mathrm{sen}\, x\cos x}dx = \ln\left|\tan\frac{x}{2}\right| - \ln\left|1-\tan\frac{x}{2}\right| + \ln\left|1+\tan\frac{x}{2}\right|$$

*Realizamos la segunda integral:

$$\int \frac{\mathrm{sen}\, x}{\cos x}dx = -ln|cosx|$$

*Por tanto:

$$\int \frac{(1+(\operatorname{sen} x)^2)}{\operatorname{sen} x \cos x} dx = \ln\left|\tan \frac{x}{2}\right| - \ln\left|1 - \tan \frac{x}{2}\right| + \ln\left|1 + \tan \frac{x}{2}\right| - ln|cosx| + C =$$

o) $\int \frac{dx}{1+\operatorname{sen}^2 x}$

$t = tgx$	$\operatorname{sen} x = \frac{t}{\sqrt{t^2+1}}$	$\operatorname{sen}^2 x = \frac{t^2}{t^2+1}$
$dx = \frac{dt}{1+t^2}$	$\cos x = \frac{1}{\sqrt{t^2+1}}$	$\cos^2 x = \frac{1}{t^2+1}$

$$\int \frac{dx}{1+\operatorname{sen}^2 x} = \int \frac{\frac{dt}{1+t^2}}{1+\frac{t^2}{t^2+1}} = \int \frac{\frac{dt}{1+t^2}}{\frac{t^2+1+t^2}{t^2+1}} = \int \frac{dt}{2t^2+1} = \int \frac{dt}{2\left(t^2+\frac{1}{2}\right)} = \frac{1}{2}\int \frac{dt}{t^2+\frac{1}{2}} =$$

$$= \frac{1}{2} \cdot \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{\frac{1}{2}}} + C = \frac{1}{2} \cdot \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{\operatorname{tg} x}{\sqrt{\frac{1}{2}}} + C = \frac{1}{2} \cdot \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{\operatorname{tg} x}{\frac{1}{\sqrt{2}}} + C = \frac{1}{\sqrt{2}} \operatorname{arctg}\left(\sqrt{2} \cdot \operatorname{tg} x\right) + C$$

p) $\int \operatorname{sen} 5x \cdot \cos 4x \, dx$

$$\operatorname{sen} \alpha \cdot \cos \beta = \frac{\operatorname{sen}(\alpha+\beta) + \operatorname{sen}(\alpha-\beta)}{2}$$

$$\int \operatorname{sen} 5x \cdot \cos 4x \, dx = \int \frac{\operatorname{sen}(5x+4x) + \operatorname{sen}(5x-4x)}{2} dx = \frac{1}{2}\int (\operatorname{sen} 9x + \operatorname{sen} x) dx =$$

$$= \frac{1}{2}\left(\int \operatorname{sen} 9x \, dx + \int \operatorname{sen} x \, dx\right) = \frac{1}{2}\left(\frac{1}{9}\int \operatorname{sen} 9x \cdot 9 \, dx + \int \operatorname{sen} x \, dx\right) =$$

$$= \frac{1}{2}\left(\frac{1}{9} \cdot (-\cos 9x) + (-\cos x)\right) + C = -\frac{1}{2}\left(\frac{\cos 9x}{9} + \cos x\right) + C$$

q) $\int \frac{dx}{13 + 12 \cos x}$

$t = \operatorname{tg} \frac{x}{2}$	$\frac{x}{2} = \operatorname{arctg} t \rightarrow$	$x = 2 \operatorname{arctg} t$	$dx = \frac{2}{1+t^2} dt$
$\cos x = \frac{1-t^2}{1+t^2}$		$\operatorname{sen} x = \frac{2t}{1+t^2}$	$\operatorname{tg} x = \frac{2t}{1-t^2}$

$$\int \frac{dx}{13+12\cos x} = \int \frac{\frac{2dt}{1+t^2}}{13+12\left(\frac{1-t^2}{1+t^2}\right)} = \int \frac{\frac{2dt}{1+t^2}}{13+\frac{12-12t^2}{1+t^2}} = \int \frac{\frac{2dt}{1+t^2}}{\frac{13+13t^2+12-12t^2}{1+t^2}} = \int \frac{2dt}{t^2+25} = 2\int \frac{1}{t^2+25} dt =$$

$$= 2\int \frac{1}{t^2+5^2} dt = 2 \cdot \frac{1}{5} \operatorname{arctg} \frac{t}{5} + C = \frac{2}{5} \operatorname{arctg}\left(\frac{\operatorname{tg} \frac{x}{2}}{5}\right) + C$$

14. Resuelve las siguientes integrales definidas:

a) $\int_0^6 (x^2 + x + 1) dx$ $= \frac{x^3}{3} + \frac{x^2}{2} + x\Big|_0^6 = \left(\frac{6^3}{3} + \frac{6^2}{2} + 6\right) - \left(\frac{0^3}{3} + \frac{0^2}{2} + 0\right) = 92$

b) $\int_{-1}^1 (x^2 + x + 1) dx$ $= \frac{x^3}{3} + \frac{x^2}{2} + x\Big|_{-1}^1 = \left(\frac{1^3}{3} + \frac{1^2}{2} + 1\right) - \left(\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + (-1)\right) = \frac{8}{3}$

c) $\int_0^{\sqrt{3}} x\sqrt{x^2+1}\,dx$ $=\frac{1}{2}\int_0^{\sqrt{3}}2x(x^2+1)^{\frac{1}{2}}dx=\frac{1}{2}\frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^{\sqrt{3}}=\frac{1}{3}\left[\left(((\sqrt{3})^2+1)^{\frac{3}{2}}\right)-\left((0^2+1)^{\frac{3}{2}}\right)\right]=\frac{7}{3}$

d) $\int_{-1}^{1}\frac{x+1}{x^2+2x+2}dx$ $=\frac{1}{2}\int_{-1}^{1}\frac{2x+2}{x^2+2x+2}dx=\frac{1}{2}ln|x^2+2x+2|\Big|_{-1}^{1}=\frac{1}{2}[(ln5)-(ln1)]=\frac{ln5}{2}$

e) $\int_0^{\pi}\operatorname{sen}x\,dx$ $=-cosx|_0^{\pi}=-cos\pi+cos0=2$

f) $\int_1^e \ln x\,dx$ $\int lnxdx=\begin{cases} u=lnx & \rightarrow du=\frac{1}{x}dx \\ dv=dx & \rightarrow v=\int dx=x\end{cases}=xlnx-\int x\frac{1}{x}dx=xlnx-x+C$

$\int_1^e lnxdx=xlnx-x]_1^e=(elne-e)-(1ln1-1)=1$

15.

Halla el valor de c que verifica $\int_0^5(2x+1)dx=f(c)\cdot(5-0)$ y razona su interpretación geométrica.

$\int_0^5(2x+1)dx=x^2+x|_0^5=30$, f(c) = 2c + 1, 30 = (2c + 1)·5 , $c=\frac{25}{10}=\frac{5}{2}$

16. Sin efectuar el cálculo de la integral indefinida, calcula f'(x) si $f(x)=\int_2^{e^x}\frac{dt}{lnt}$

La función $g(t)=\frac{1}{lnt}$ es continua en [2,b] , $g(x)=e^x$ es derivable,

Por el teorema fundamental del cálculo integral:

$$f'(x)=\frac{1}{\ln(e^x)}\cdot e^x$$

EJERCICIOS Y PROBLEMAS

1.

Sabiendo que $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ y $\int f^n(x) f'(x) dx = \frac{f^{n+1}(x)}{n+1} + C$, calcula:

1) $\int x^5 dx = \frac{x^6}{6} + C$

2) $\int \frac{4}{x^5} dx = \int 4x^{-5} dx = 4 \cdot \frac{x^{-4}}{-4} = \frac{-1}{x^4} + C$

3) $\int \frac{dx}{x^2} = \int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} + C$

4) $\int 37 dx = 37x + C$

5) $\int 6x^7 dx = 6 \cdot \frac{x^8}{8} + C$

6) $\int 5x^{\frac{1}{4}} dx = 5 \cdot \frac{x^{\frac{5}{4}}}{\frac{5}{4}} = 4\sqrt[4]{x^5} + C$

7) $\int 5 \cdot \sqrt{x^3} dx = \int 5x^{\frac{3}{2}} dx = 5 \cdot \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = 2\sqrt{x^5} + C$

8) $\int (3 - 2x - x^4) dx = 3x - \frac{2x^2}{2} - \frac{x^5}{5} + C = 3x - x^2 - \frac{x^5}{5} + C$

9) $\int (2x^5 - 5x + 3) dx = \frac{2x^6}{6} - \frac{5x^2}{2} + 3x + C$

10) $\int (2 + 3x^3)^2 dx = \int 4 + 9x^6 + 12x^3 dx = 4x + 9\frac{x^7}{7} + 12\frac{x^4}{4} + C = 4x + \frac{9x^7}{7} + 3x^4 + C$

11) $\int (2 \cdot (x^2 + 2)^3 dx = \int 2 \cdot ((x^2)^3 + (3x^2)^2 \cdot 2 + 3x^2 \cdot 2^2 + 2^3) dx = \int 2 \cdot (x^6 + 18x^4 + 12x^2 + 8) dx =$

$\int (2x^6 + 36x^4 + 24x^2 + 16) dx = 2 \cdot \frac{x^7}{7} + 36 \cdot \frac{x^5}{5} + 24 \cdot \frac{x^3}{3} + 16x + C$

12) $\int (1 - x^3)^2 dx = \int 1 - 2x^3 + (x^3)^2 dx = x - 2 \cdot \frac{x^4}{4} + \frac{x^7}{7} + C$

13) $\int \frac{x^3 - x + 2}{x^3} dx = \int \frac{\cancel{x^3}}{\cancel{x^3}} dx - \int \frac{\cancel{x}}{x^{\cancel{2}}} dx + \int \frac{2}{x^3} dx = \int 1 dx - \int \frac{1}{x^2} dx + \int \frac{2}{x^3} dx = \int (1 - x^{-2} + 2x^{-3}) dx =$

$= x + \frac{x^{-1}}{1} + 2 \cdot \frac{x^{-2}}{-2} = x + \frac{x^{-1}}{1} - x^{-2} + C$

14) $\int (-4x^{\frac{2}{3}} + 2x) dx = -4 \cdot \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \cancel{2} \cdot \frac{x^2}{\cancel{2}} + C = -\frac{12X^{\frac{5}{3}}}{5} + x^2 + C$

15) $\int (3a - \frac{1}{3e^2} + 2x^a) dx = (3a - \frac{1}{3e^2})x + 2\frac{x^{a+1}}{a+1} + C$

16) $\int -\frac{3}{x^3} + 2 - \frac{3}{\sqrt{x}} dx = \int -3x^{-3} + 2 - 3x^{\frac{-1}{2}} dx = -3\frac{x^{-2}}{-2} + 2x - 3\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C =$

$= \frac{3}{2x^2} + 2x - 6\sqrt{x} + C$

17) $\int(3x^5 - \frac{4}{3x^2} + 2\sqrt[5]{x^2})dx = \frac{3x^6}{6} - 12\cdot\frac{x^{-1}}{-1} + 2\cdot\frac{x^{\frac{7}{5}}}{\frac{7}{5}} = \frac{x^6}{2} - \frac{12}{x} + \frac{10x^{\frac{7}{5}}}{7} + C$

18) $\int(1-x)\sqrt{x}dx = \int\sqrt{x} - x\sqrt{x}dx = \int x^{\frac{1}{2}} - x\cdot x^{\frac{1}{2}}\,dx = \int x^{\frac{1}{2}} - x^{\frac{3}{2}}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2x^{\frac{3}{2}}}{3} - \frac{2x^{\frac{5}{2}}}{5} + C$

19) $\int\frac{x^3+5x^2-4}{x^2}dx = \int\frac{x^3}{x^2}dx + \int\frac{5x^2}{x^2}dx - \int\frac{4}{x^2}dx = \int x\,dx + \int 5\,dx - \int 4x^{-2}dx =$

$= \frac{x^2}{2} + 5x - 4\cdot\frac{x^{-1}}{-1} = \frac{x^2}{2} + 5x - \frac{4}{x} + C$

20) $\int(5e^x + \frac{2x^3-3x^2+5}{4x^2})dx = \int 5e^x dx + \int\frac{2x^3}{4x^2}dx - \int\frac{3x^2}{4x^2}dx + \int\frac{5}{4x^2}dx =$

$= \int 5e^x dx + \int\frac{2x}{4}dx - \int\frac{3}{4}dx + \int\frac{5x^{-2}}{4}dx \quad = 5e^x + \frac{1}{4}x^2 - \frac{3}{4}x - \frac{5x^{-1}}{4} =$

$= 5e^x + \frac{1}{4}x^2 - \frac{3}{4}x + \frac{5}{4x} + C$

21) $\int\frac{(1+x)^2}{\sqrt{x}}dx \quad = \int(x^2+2x+1)\left(x^{\frac{-1}{2}}\right)dx = \int\left(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + x^{\frac{-1}{2}}\right)dx = \frac{x^{5/2}}{5/2} + 2\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \frac{2\sqrt{x^5}}{5} + \frac{4\sqrt{x^3}}{3} + 2\sqrt{x} + C$

22) $\int(\sqrt{x} - \frac{1}{2}x + \frac{2}{\sqrt{x}})dx = \int\left(x^{\frac{1}{2}} - \frac{1}{2}x + 2x^{\frac{-1}{2}}\right)dx = \frac{x^{3/2}}{3/2} - \frac{x^2}{4} + \frac{2x^{1/2}}{1/2} + C = \frac{2\sqrt{x^3}}{3} - \frac{x^2}{4} + 4\sqrt{x} + C$

23) $\int\sqrt{x}(x^3+1)dx = \int\left(x^{\frac{1}{2}}\right)(x^3+1)dx = \int\left(x^{\frac{7}{2}}\right) + \left(x^{\frac{1}{2}}\right)dx = \frac{x^{9/2}}{9/2} + \frac{x^{3/2}}{3/2} + C =$

$= \frac{2\sqrt{x^9}}{9} + \frac{2\sqrt{x^3}}{3} + C$

24) $\int\left(\sqrt{x^5} - \frac{2}{3\sqrt{x}}\right)dx = \int\left(x^{\frac{5}{2}} - \frac{2x^{\frac{-1}{2}}}{3}\right)dx = \frac{x^{7/2}}{7/2} - \frac{2x^{1/2}}{3\cdot 1/2} + C = \frac{2\sqrt{x^7}}{7} - \frac{4\sqrt{x}}{3} + C$

25) $\int\sqrt{x}(3-5x)dx = \int(x^{\frac{1}{2}})(3-5x)dx = \int\left(3x^{\frac{1}{2}} - 5x^{\frac{3}{2}}\right)dx = \frac{3x^{3/2}}{3/2} - \frac{5x^{5/2}}{5/2} + C =$

$= 2\sqrt{x^3} - 2\sqrt{x^5} + C$

26) $\int\frac{(x+1)+(x-2)}{\sqrt{x}}dx \qquad \int(x^2+x-2)\left(x^{\frac{-1}{2}}\right)dx = \int\left(x^{\frac{3}{2}} + x^{\frac{1}{2}} - 2x^{\frac{-1}{2}}\right)dx = \frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} - \frac{2x^{1/2}}{1/2} + C = \frac{2\sqrt{x^5}}{5} + \frac{2\sqrt{x^3}}{3} - 4\sqrt{x} + C$

27) $\int(3x+4)^2dx = \int 3(3x+4)^2dx = \frac{(3x+4)^3}{3\cdot 3} + C = \frac{(3x+4)^3}{9} + C$

28) $\int(3x-7)^4dx = \frac{1}{3}\int 3(3x-7)^4dx = \frac{(3x-7)^5}{3\cdot 5} + C = \frac{(3x-7)^5}{15} + C$

29) $\int x(x^2-4)^3dx = \frac{1}{2}\int 2x(x^2-4)^3dx = \frac{(x^2-4)^4}{2\cdot 4} + C = \frac{(x^2-4)^4}{8} + C$

30. $\int 3x\,(x^2+2)^3\,dx = \frac{3}{2}\int 2x\,(x^2+2)^3 dx = \frac{3(x^2+2)^4}{2\cdot 4} + C = \frac{3(x^2+2)^4}{8} + C$

31. $\int (x^3+2)^2 x^2\,dx = \frac{1}{3}\int (x^3+2)^2 3x^2 dx = \frac{(x^3+2)^3}{3\cdot 3} + C = \frac{(x^3+2)^3}{9} + C$

32. $\int (x^3+3)\,x^2\,dx = \frac{1}{3}\int (x^3+3)3x^2 dx = \frac{(x^3+3)^2}{3\cdot 2} + C = \frac{(x^3+3)^2}{6} + C$

33. $\int (x-2)^{3/2} dx = \frac{2\sqrt{(x-2)^5}}{5} + C$

34. $\int (a + x)^3\,dx = \frac{(a+x)^4}{4} + C$

35. $\int [(x+2)^3 - (x+2)^2]\,dx = \frac{(x+2)^4}{4} - \frac{(x+2)^3}{3} + C$

36. $\int \sqrt{3x+12}\,dx = \frac{1}{3}\int 3(3x+12)^{\frac{1}{2}} dx = \frac{(3x+12)^{3/2}}{3\cdot 3/2} + C = \frac{2\sqrt{(3x+12)^3}}{9} + C$

37. $\int \frac{dx}{\sqrt{x+3}} = \int (x+3)^{\frac{-1}{2}} dx = \frac{(x+3)^{1/2}}{1/2} + C = 2\sqrt{x+3} + C$

38. $\int \frac{dx}{(x-1)^3} = \int (x-1)^{-3} dx = \frac{(x-1)^{-2}}{-2} + C = \frac{-1}{2(x-1)^2} + C$

39. $\int (x^2+x)^4\,(2x+1)\,dx = \frac{(x^2-x)^5}{5} + C$

40. $\int \frac{1}{\sqrt{x}}(1+\sqrt{x})^2 dx = 2\int \frac{1}{2\sqrt{x}}\left(1+\sqrt{x}\right)^2 dx = \frac{2(1+\sqrt{x})^3}{3} + C$

41. $\int \frac{x^3}{(x^4-1)^2}\,dx = \int x^3(x^4-1)^{-2}\,dx = \frac{1}{4}\int 4x^3(x^4-1)^{-2}\,dx = \frac{1}{4}\cdot\frac{(x^4-1)^{-1}}{-1} + c = \frac{(x^4-1)^{-1}}{-4} + c = \frac{1}{4(x^4-1)} + c$

42. $\int \frac{x}{(x^2+4)^3}\,dx = \int x(x^2+4)^{-3} dx = \frac{1}{2}\int 2x(x^2+4)^{-3} dx = \frac{1}{2}\cdot\frac{(x^2+4)^{-2}}{-2} + c = \frac{(x^2+4)^{-2}}{-4} + c = -\frac{1}{4(x^2+4)^2} + c$

43. $\int x\sqrt{x^2-7}\,dx = \int x(x^2-7)^{\frac{1}{2}}\,dx = \frac{1}{2}\int 2x\,(x^2-7)^{\frac{1}{2}}\,dx = \frac{1}{2}\cdot\frac{(x^2-7)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{(x^2-7)^{\frac{3}{2}}}{3} + c = \frac{\sqrt{(x^2-7)^3}}{3} + c$

44. $\int (x-1)(x^2-2x+3)^4\,dx =$

$\frac{1}{2}\int (2x-2)(x^2-2x+3)^4\,dx = \frac{1}{2}\cdot\frac{(x^2-2x+3)^5}{5} + c = \frac{(x^2-2x+3)^5}{10} + c$

45. $\int \frac{3x}{\sqrt{1+7x^2}}\,dx = \int 3x(1+7x^2)^{-\frac{1}{2}}\,dx =$

$\frac{3}{14}\int 14x(1+7x^2)^{-\frac{1}{2}}\,dx = \frac{3}{14}\cdot\frac{(1+7x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{3\sqrt{1+7x^2}}{7} + c$

46. $\int \frac{8x^2}{(x^3+2)^2} dx = \int 8x^2\,(x^3+2)^{-2}dx =$

$\frac{8}{3}\int 3x^2\,(x^3+2)^{-2}dx = \frac{8}{3}\cdot\frac{(x^3+2)^{-1}}{-1} + c = -\frac{8}{3(x^3+2)} + c$

47. $\int \frac{3x}{\sqrt[3]{x^2+3}} dx = \int 3x(x^2+3)^{-\frac{1}{3}}\,dx = \frac{3}{2}\int 2x(x^2+3)^{-\frac{1}{3}}\,dx =$

$\frac{3}{2}\cdot\frac{(x^2+3)^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{9\sqrt[3]{(x^2+3)^2}}{4} + c$

48. $\int x\sqrt[3]{1-x^2}\,dx = \int x(1-x^2)^{\frac{1}{3}}\,dx = \frac{1}{2}\int 2x(1-x^2)^{\frac{1}{3}}\,dx =$

$\frac{1}{2}\cdot\frac{(1-x^2)^{\frac{4}{3}}}{\frac{4}{3}} + c = \frac{3\sqrt[3]{(1-x^2)^4}}{4} + c$

49. $\int \frac{x^2}{\sqrt[4]{x^3+5}} dx = \int x^2(x^3+5)^{-\frac{1}{4}}\,dx = \frac{1}{3}\int 3x^2(x^2+5)^{-\frac{1}{4}}\,dx = \frac{1}{3}\cdot\frac{(x^3+5)^{\frac{3}{4}}}{\frac{3}{4}} + c = \frac{4\sqrt[4]{(x^3+5)^3}}{9} + c$

50. $\int x^2(x^3-1)^{\frac{3}{5}}\,dx = \frac{1}{3}\int 3x^2(x^3-1)^{\frac{3}{5}}dx =$

$\frac{1}{3}\cdot\frac{(x^3-1)^{\frac{3}{5}}}{\frac{3}{5}} + c = \frac{5\sqrt[5]{(x^3-1)^8}}{24} + c$

51. $\int \sqrt{x^2-2x^4}\,dx = \int \sqrt{x^2(1-2x^2)}\,dx = \int x\sqrt{1-2x^2}dx = \int x(1-2x^2)^{\frac{1}{2}}\,dx = -\frac{1}{4}\int -4x(1-2x^2)^{\frac{1}{2}}\,dx =$

$-\frac{1}{4}\cdot\frac{(1-2x^2)^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{\sqrt{(1-2x^2)^3}}{6} + c$

52. $\int (e^x+1)^3e^x dx = \frac{(e^x+1)^4}{4} + c$

53. $\int sen^3x(\cos x)\,dx = \frac{sen^4x}{4} + c$

54. $\int x(\cos^4x^2)sen\,x^2\,dx = -\frac{1}{2}\int 2x\,(\cos^4x^2)(-sen\,x^2)dx = -\frac{1}{2}\cdot\frac{\cos^5x^2}{5} + c = -\frac{\cos^5x^2}{10} + c$

55. $\int \frac{x\cdot ln(x^2+3)}{x^2+3}\,dx = \int ln\,(x^2+3)\cdot\frac{x}{x^2+3}dx =$

$\frac{1}{2}\int \ln(x^2+3)\cdot\frac{2x}{x^2+3}dx = \frac{1}{2}\cdot\frac{ln^2|x^2+3|}{2} + c = \frac{ln^2|x^2+3|}{4} + c$

56. $\int \frac{sen\,x}{\cos^3x} dx = \int \cos^{-3}x(sen\,x)dx =$

$-1\int \cos^{-3}x(-sen\,x)dx = -1\cdot\frac{\cos^{-2}x}{-2} + c = \frac{\cos^{-2}x}{2} + c = \frac{1}{2\cos^2x} + c$

57. $\int \frac{e^x}{2e^x-3} dx = \frac{1}{2}\int \frac{2e^x}{2e^x-3}dx = \frac{1}{2}\cdot ln|2e^x-3| + c$

58. $\int tg^5 x(sec^2 x)dx = \frac{tg^6 x}{6} + c$

59. $\int \frac{sec^2 3x}{tg\, 3x}dx = \frac{1}{3}\int \frac{3\cdot sec^2 3x}{tg\, 3x}dx = \frac{1}{3}\cdot ln\,|tg\, 3x| + c$

60. $\int \frac{ln(x)}{3x}dx = \frac{1}{3}\int ln\,|x|\cdot\frac{1}{x}dx = \frac{1}{3}\cdot\frac{ln^2|x|}{2} + c = \frac{ln^2|x|}{6} + c$

2. Sabiendo que

$$\int \frac{1}{x}dx = \ln|x| + C \text{ y } \int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C \int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C\text{, calcula:}$$

1) $\int \frac{dx}{x+2} = \int \frac{1}{x+2}dx = \ln|x+2| + C$

2) $\int \frac{dx}{2x-3} = \frac{1}{2}\ln|2x-3| + C$

3) $\int \frac{dx}{x-1} = \int \frac{1}{x-1}dx = \ln|x-1| + C$

4) $\int \frac{xdx}{x^2-1} = \frac{1}{2}\int \frac{2x}{x^2-1}dx = \frac{1}{2}\ln|x^2-1| + C$

5) $\int \frac{x}{1-2x^3}dx = \frac{-1}{6}\int \frac{-6x^2}{1-2x^3} = \frac{-1}{6}\ln|2x^3-1| + C$

6) $\int \frac{x^2}{1-x^3}dx = \frac{-1}{3}\int \frac{3x^2}{1-x^3}dx = \frac{-1}{3}\ln|x^3-1| + C$

7) $\int \frac{3x}{x^2+2}dx = \frac{3}{2}\int \frac{2x}{x^2+2}dx = \frac{3}{2}\ln|x^2+2| + C$

8) $\int \frac{4}{3x+5}dx = \frac{4}{3}\int \frac{3}{3x+5}dx = \frac{4}{3}\ln|3x+5| + C$

9) $\int \frac{x+1}{x^2+2x+2}dx = \frac{1}{2}\int \frac{2(x+1)}{x^2+2x+2}dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x+2}dx = \frac{1}{2}\ln|x^2+2x+2| + C$

10) $\int \left(\sqrt{x} + \frac{1}{x}\right)dx = \int \sqrt{x}dx + \int \frac{1}{x}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \ln|x| + C$

11) $\int \left(\frac{3}{x^2} + \frac{2}{x} + \sqrt{x}\right)dx = \int 3x^{-2} + \frac{2}{x} + x^{\frac{1}{2}}dx = \frac{3x^{-1}}{-1} + 2\ln(x) + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = -3x^{-1} + 2\ln(x) + \frac{2\sqrt{x^3}}{3} + C$

12) $\int \frac{dx}{x\ln x} = \ln(|\ln|x||) + C$

13) $\int \frac{dx}{\sqrt{x}(1-\sqrt{x})} = \int \frac{1}{\sqrt{x}(1-\sqrt{x})}dx = -2\int \frac{-1}{2\sqrt{x}(1-\sqrt{x})}dx = -\,2\ln(|1-\sqrt{x}|) + C$

14) $\int \frac{1}{2x-1} - \frac{1}{2x+1}dx = \frac{1}{2}(\int \frac{2}{(2x-1)}dx - \int \frac{2}{(2x-1)}dx) = \frac{1}{2}\ln(|2x-1|) - \frac{1}{2}\ln(|2x+1|) + C$

15) $\int \frac{\mathbf{e^x}}{\mathbf{e^x+1}}\mathbf{dx} = \ln(e^x+1)+C$

16) $\int \frac{\mathbf{e^{2x}}}{\mathbf{e^{2x}+3}}\mathbf{dx} = \frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+3}dx = \frac{1}{2}\ln(e^{2x}+3)+C$

17) $\int \mathbf{tan(x)\,dx} = \int \frac{\sin(x)}{\cos(x)}dx = -\ln(|\cos(x)|)+C$

18) $\int \frac{\mathbf{cos(x)}}{\mathbf{sin(x)}}\mathbf{dx} = \ln(|\sin(x)|)+C$

19) $\int \frac{\mathbf{5}}{\mathbf{xln(x)}}\mathbf{dx} = 5\ln(|\ln(x)|)+C$

20) $\int \frac{\mathbf{sin(x)+cos(x)}}{\mathbf{cos(x)}}\mathbf{dx} = \int\left(\frac{\sin(x)}{\cos(x)}+\frac{\cos(x)}{\cos(x)}\right)dx = \int\left(\frac{\sin(x)}{\cos(x)}+1\right)dx = \int\frac{\sin(x)}{\cos(x)}dx+\int 1dx =$
$-\ln(|\cos(x)|)+x+C$

21) $\int \frac{\mathbf{2\,sin(x)\,cos(x)}}{\mathbf{1+sin(x^2)}}\mathbf{dx} = \ln(|1+\sin(x^2)|)+C$

22) $\int \frac{\mathbf{sin(x)-cos(x)}}{\mathbf{sin(x)+cos(x)}}\mathbf{dx} = -\ln(|\sin(x)+\cos(x)|)+C$

23) $\int \mathbf{x\cot x^2\,dx} = \frac{1}{2}\int 2x\frac{\cos(x^2)}{\sin(x^2)}dx = \frac{\ln(|\sin x^2|)}{2}+C$

3. Si $\int e^x\,dx = e^x + C,\quad \int e^{f(x)}\cdot f'(x)dx = e^{f(x)}+C,$

$\int a^x dx = \frac{a^x}{\ln a}+C$ **y** $\int a^{f(x)}\cdot f'(x)dx = \frac{a^{f(x)}}{\ln a}+C,$ **calcula:**

1. $\int 3^x\,dx = \frac{3^x}{\ln 3}+C$
2. $\int a^{4x}dx = \frac{1}{4}\int 4(a^{4x})dx = \frac{a^{4x}}{4\ln a}+C$
3. $\int e^{-x}dx = -1\int(-1)(e^{-x})dx = \frac{-1}{e^x}+C$
4. $\int 4e^{3x}dx = 4(\frac{1}{3})\int 3(e^{3x})dx = \frac{4e^{3x}}{3}+C$
5. $\int(3x^2\cdot e^{x^3+2})\,dx = e^{x^3+2}+C$
6. $\int(4e^{4-x})dx = 4\cdot(-1)\int(-1)(e^{4-x})dx = -4(e^{4-x})+C$
7. $\int(x^2e^{x^3})\,dx = \frac{1}{3}\int[3x^2(e^{x^3})]dx = \frac{e^{x^3}}{3}+C$
8. $\int(e^x+1)^2\,dx = \int[(e^x)^2+2(e^x)(1)+1^2]dx = \int e^{2x}dx+\int 2e^xdx+\int 1dx =$
   $=\frac{1}{2}\int 2e^{2x}dx+\int 2e^xdx+\int 1dx = \frac{e^{2x}}{2}+2e^x+x+C$
9. $\int\left(e^x+\frac{1}{e^x}\right)^2dx = \int(e^x+e^{-x})^2dx = \int[(e^x)^2+2(e^x)(e^{-x})+(e^{-x})^2]dx$

IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

$= \int e^{2x}\,dx + \int 2dx + \int e^{-2x}dx = \frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2} + C$

10. $\int (e^x + x^6)^2 dx = \int [(e^x)^2 + 2(e^x)(x^6) + (x^6)^2]dx =$

$= \frac{1}{2}\int 2e^{2x}\,dx + \int 2(e^x)(x^6)\,dx\ (por\ partes) + \int x^{12}dx =$

$= \frac{1}{2}e^{2x} + 2e^x(x^6 - 6x^5 + 30x^4 - 120x^3 + 360x^2 - 720x + 720\ ) + \frac{x^{13}}{13} + C$ -

11. $\int e^{-x^2+2}\,xdx = \frac{-1}{2}\int e^{-x^2+2}\,(-2x)dx = \frac{-\left(e^{-x^2+2}\right)}{2} + C$

12. $\int \frac{e^{\ln x}}{x}dx = \int \frac{x}{x}dx = \int dx = x + C$

13. $\int \frac{e^{\frac{1}{x^2}}}{x^3}dx = \int e^{\frac{1}{x^2}} \cdot \frac{1}{x^3}dx = \frac{-1}{2}\int e^{\frac{1}{x^2}} \cdot \frac{-2}{x^3}dx = \frac{-1}{2}e^{\frac{1}{x^2}} + C$

14. $\int xe^{\sin x^2}\cos x^2\,dx = \frac{1}{2}\int 2x\,e^{\sin x^2}\cos x^2\,dx = \frac{e^{\sin x^2}}{2} + C$

15. $\int (e^{3\cos 2x}\sin 2x)dx = \frac{-1}{6}\int (-6)\,(e^{3\cos 2x}\sin 2x)dx = \frac{-e^{3\cos 2x}}{6} + C$

16. $\int \frac{e^{\sqrt{x}}}{5\sqrt{x}}dx = \frac{1}{5}\cdot 2\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}dx = \frac{2e^{\sqrt{x}}}{5} + C$

17. $\int e^{\cos x}\sin x\,dx = -\int e^{\cos x}\,(-\sin x)\,dx = -e^{\cos x} + C$

18. $\int \left(\frac{\sqrt{1+e^2}}{2e} - e^{x-3}\right)dx = \frac{\sqrt{1+e^2}}{2e}x - e^{x-3} + C$

19. $\int e^{\tan 2x}\sec^2 2x\,dx = \frac{1}{2}\int 2e^{\tan 2x}\sec^2 2x\,dx = \frac{e^{\tan 2x}}{2} + C$

20. $\int \frac{2x}{3}(3^{3+5x^2})dx = \frac{2}{3}\int x(3^{3+5x^2})dx = \frac{2}{3}\left(\frac{1}{10}\right)\int 10x(3^{3+5x^2})dx = \frac{1}{15}\left(\frac{3^{3+5x^2}}{\ln 3}\right) + C$

$$\int \frac{x}{2}(2^{3-5x^2})dx = \frac{1}{2}\int x(2^{3-5x^2})dx = \frac{1}{2}\cdot(-\frac{1}{10})\int (-10x)(2^{3-5x^2})dx = \frac{-1}{20}(\frac{2^{3-5x^2}}{\ln 2}) + C$$

4. Sabiendo que $\int \sin x\,dx = -\cos x + C,\quad \int f'(x)\cdot \sin f(x) = -\cos f(x) + C,$

$\int \cos x dx = \sin x + C \quad y \quad \int \cos f(x)\cdot f'(x) = \sin f(x) + C$ **calcula:**

1. $\int \sin(2x+8)dx = \frac{1}{2}\int \sin(2x+8)2dx = \frac{-\cos(2x+8)}{2} + C$

2. $\int \sin\frac{x}{2}dx = 2\int \sin(\frac{x}{2})(\frac{1}{2})dx = -2\cos\frac{x}{2} + C$

3. $\int \cos 3x\,dx = \frac{1}{3}\int 3(\cos(3x)dx) = \frac{\sin 3x}{3} + C$

4. $\int x\sin x^2 dx = \frac{1}{2}\int 2x(\sin x^2)\,dx = \frac{-\cos x^2}{2} + C$

5. $\int (\frac{3\sin x - 2\cos x}{4})dx = \frac{1}{4}\int (3\sin x - 2\cos x)dx = \frac{-3\cos x}{4} - \frac{\sin x}{2} + C$

6. $\int \sin 2x\,dx = \frac{1}{2}\int 2\,(\sin 2x)dx = \frac{-\cos 2x}{2} + C$

7. $\int e^x \cos e^x \, dx = \sin e^x + C$

8. $\int x\cos(2x^2)\cdot\sin(2x^2)dx = \frac{1}{4}\int 4x\cos(2x^2)\cdot\sin(2x^2)dx = -\frac{1}{4}\cos(2x^2) + C$

9. $\int \frac{\sin(\ln x)}{x}dx = \int \frac{1}{x}\cdot\sin(lnx)\,dx = -\cos(lnx) + C$

7. Si

$$\int \frac{1}{\cos^2 x}dx = \int\left(1+\mathrm{tg}^2 x\right)dx = \mathrm{tg}\, x + C$$ y $$\int \frac{f'(x)}{\cos^2 f(x)}dx = \int\left[1+\mathrm{tg}^2 f(x)\right]\cdot f'(x)dx = \mathrm{tg}\, f(x) + C$$, calcula:

1) $\int x(1+\tan x^2)\,dx = \frac{1}{2}\int 2x(1+\tan x^2)\,dx = \frac{1}{2}tan(x^2) + \mathrm{C}$

2) $\int(1+\tan x)^2\,dx = \int(1+tan^2x + 2\tan x)dx = \int((1+tan^2x)+2\tan x)dx = \tan x - 2\int\frac{-\sin x}{\cos x}dx = \tan x - 2\ln|\cos x| + \mathrm{C}$

3) $\int tan^2\,3x\,dx = \int(1-1+tan^2\,3x\,)dx = \frac{1}{3}\int 3(1+tan^2\,3x)\,dx - \int 1\,dx =$

$= \frac{1}{3}\cdot\tan 3x - x + \mathrm{C}$

6. Halla el valor de las siguientes integrales, usando un cambio de variable:

1) $\int(2+5x)^4dx = \int t^4\cdot\frac{1}{5}dt = \frac{1}{5}\int t^4dt = \frac{1}{5}\cdot\frac{t^5}{5} = \frac{t^5}{25} = \frac{(2+5x)^5}{25} + c$

t = 2+5x , dt = 5 dx $\to dx = \frac{1}{5}dt$

2) $\int(3+4x)^6dx = \int t^6\cdot\frac{1}{4}dt = \frac{1}{4}\int t^6dt = \frac{1}{4}\cdot\frac{t^7}{7} = \frac{t^7}{28} = \frac{(3+4x)^7}{28} + c$

t = 3+4x , dt = 4x dx $\to dx = \frac{1}{4}dt$

3) $\int 6x(3+x^2)^5dx = \int 6x\cdot t^5\cdot\frac{1}{2x}dt = \int 3t^5dt = 3\int t^5dt = \frac{3t^6}{6} = \frac{t^6}{2} = \frac{(3+x^2)^6}{2} + c$

t $=3+x^2$, dt = 2x dx $\to dx = \frac{1}{2x}dt$

4) $\int\left[\frac{3}{5+4x} + \frac{3}{(5+4x)^3}\right]dx = \int\left[\frac{3}{t}\cdot\frac{1}{4}\right]dt + \int\left[\frac{3}{t^3}\cdot\frac{1}{4}\right]dt = \int\left[\frac{3}{4t}\right]dt + \int\left[\frac{3}{4t^3}\right]dt =$

$\frac{3}{4}\int\left[\frac{1}{t}\right]dt + \frac{3}{4}\int\left[\frac{1}{t^3}\right] = \frac{3}{4}\int\left[\frac{1}{t}\right]dt + \frac{3}{4}\int\left[t^{-3}\right]dt = \frac{3}{4}\ln|t| + \frac{3}{4}\cdot\frac{t^{-2}}{-2} = \frac{3}{4}\ln|t| - \frac{3t^{-2}}{8} =$

$\frac{3}{4}\ln|t| - \frac{3}{8t^2} = \frac{3}{4}\ln|5+4x| - \frac{3}{8(5+4x)^2} + c$

t = 5+4x , dt = 4 dx→$dx=\frac{1}{4}dt$

5) $\int(\sqrt{3+2x}+\sqrt[3]{3+2x})dx$=$\int(\sqrt{t}+\sqrt[3]{t})\cdot\frac{1}{2}dt$=$\frac{1}{2}\int(t^{\frac{1}{2}}+t^{\frac{1}{3}})dt$=$\frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\frac{1}{2}\cdot\frac{t^{\frac{4}{3}}}{\frac{4}{3}}$=

$=\frac{t^{\frac{3}{2}}}{3}+\frac{3t^{\frac{4}{3}}}{8}$=$\frac{\sqrt{t^3}}{3}+\frac{3\sqrt[3]{t^4}}{8}$=$\frac{\sqrt{(3+2x)^3}}{3}+\frac{3\sqrt[3]{(3+2x)^4}}{8}+c$

t = 3+2x , dt = 2 dx→$dx=\frac{1}{2}dt$

6) $\int(\frac{e^x-4}{e^{2x}})dx$=$\int(\frac{t-4}{t^2})\cdot\frac{1}{t}dt$=$\int(\frac{t-4}{t^3})dt$=$\int(\frac{t}{t^3})dt-\int(\frac{4}{t^3})dt$=

$=\int(\frac{1}{t^2})dt-4\int(\frac{1}{t^3})dt$=$\int t^{-2}dt-4\int t^{-3}dt$=$\frac{t^{-1}}{-1}-\frac{4t^{-2}}{-2}$=$\frac{-1}{t}+\frac{2}{t^2}$=$\frac{-1}{e^x}+\frac{2}{e^{2x}}+c$

$t=e^x$, $dt=e^x dx$→$dx=\frac{1}{e^x}dt=\frac{1}{t}dt$

7) $\int\sin^3(x)\cdot\cos(x)dx$=$\int t^3\cdot\cos(x)\cdot\frac{1}{\cos(x)}dt$=$\int t^3 dt$=$\frac{t^4}{4}$=$\frac{\sin^4(x)}{4}+c$

$t=\sin(x)$, $dt=\cos(x)dx$→$dx=\frac{1}{\cos(x)}dt$

8) $\int(\frac{\sin(x)}{\cos(x)})dx$=$\int(\frac{\sin(x)}{t})\cdot\frac{-1}{\sin(x)}dt$=$\int\frac{-1}{t}dt=-\ln|t|$=$-\ln|\cos(x)|+c$

$t=\cos(x)$, $dt=-\sin(x)dx$→$dx=\frac{-1}{\sin(x)}dt$

9) $\int(\frac{\cos(x)}{\sin^4(x)})dx$=$\int(\frac{\cos(x)}{t^4})\cdot\frac{1}{\cos(X)}dt$=$\int(\frac{1}{t^4})dt$=$\int t^{-4}dt$=$\frac{t^{-3}}{-3}$=$\frac{-1}{3t^3}$=$\frac{-1}{3\sin^3(x)}+c$

$t=\sin(x)$, $dt=\cos(x)dx$→$dx=\frac{1}{\cos(x)}dt$

10) $\int x\sqrt{x^2+4}dx$=$\int x\sqrt{t}\cdot\frac{1}{2x}dt$=$\int(\frac{\sqrt{t}}{2})dt$=$\int(\frac{t^{\frac{1}{2}}}{2})dt$=$\frac{t^{\frac{3}{2}}}{\frac{3}{2}\cdot 2}$=$\frac{2t^{\frac{3}{2}}}{6}$=$\frac{t^{\frac{3}{2}}}{3}$=$\frac{\sqrt[3]{(x^2+4)^2}}{3}+c$

$t=x^2+4$, dt = 2x dx→$dx=\frac{1}{2x}dt$

11) $\int(\frac{e^x+3}{e^{2x}})dx$=$\int(\frac{t+3}{t^2}\cdot\frac{1}{t})dt$=$\int(\frac{t+3}{t^3})dt$=$\int(\frac{t}{t^3})dt+\int(\frac{3}{t^3})dt$=

$\int t\cdot t^{-3}dt+\int 3t^{-3}dt$=$\int t^{-2}dt+\int 3t^{-3}dt$=$\frac{t^{-1}}{-1}+\frac{3t^{-2}}{-2}$=$\frac{-1}{t}-\frac{3}{2t^2}$=$\frac{-1}{e^x}-\frac{3}{2e^{2x}}+c$

IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

$t = e^x \quad , \quad dt = e^x dx \rightarrow dx = \frac{1}{e^x} dt = \frac{1}{t} dt$

12) $\int \left(\frac{e^{-x}+2}{e^{3x}}\right) dx = \int \left(\frac{-t+2}{t^3} \cdot \frac{1}{t}\right) dt = \int \left(\frac{-t+2}{t^4}\right) dt = \int \left(\frac{-t}{t^4}\right) dt + \int \left(\frac{2}{t^4}\right) dt =$

$\int (-t) \cdot t^{-4} dt + \int 2t^{-4} dt = \int (-t^{-3}) dt + \int 2t^{-4} dt = \frac{t^{-2}}{2} - \frac{2t^{-3}}{3} = \frac{e^{-2x}}{2} - \frac{2e^{-3x}}{3} + c$

$t = e^x \quad , \quad dt = e^x dx \rightarrow dx = \frac{1}{e^x} dt = \frac{1}{t} dt$

13) $\int \frac{(x-2\sqrt{x})^2}{3x^2} \mathbf{dx}$

$\int \frac{(x-2\sqrt{x})^2}{3x^2} dx = \frac{1}{3} \int \left(\frac{x^2-4\sqrt{x}+4x}{x^2}\right) dx = \frac{1}{3} \int \left(\frac{x^2}{x^2} - \frac{4\sqrt{x}}{x^2} + \frac{4x}{x^2}\right) dx = \frac{1}{3}\left(\int 1dx - 4\int x^{-\frac{3}{2}} dx + 4\int \frac{1}{x} dx\right) =$

$= \frac{1}{3}\left(x - 4\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 4ln|x|\right) + C = \frac{1}{3}\left(x + \frac{8}{\sqrt{x}} + 4ln|x|\right) + C$

14) $\int \frac{(2+3\sqrt{x})^2}{4x} \mathbf{dx}$

$\int \frac{(2+3\sqrt{x})^2}{4x} dx = \frac{1}{4} \int \left(\frac{2^2+12\sqrt{x}+9x}{x}\right) dx = \frac{1}{4} \int \left(\frac{4}{x} - \frac{12\sqrt{x}}{x} + \frac{9x}{x}\right) dx = \frac{1}{4}\left(\int \frac{4}{x} dx + 12\int x^{-\frac{1}{2}} dx + \int 9dx\right) =$

$= \frac{1}{4}\left(4ln|x| + 12\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 9x\right) + C = ln|x| + 6\sqrt{x} + \frac{9x}{4} + C$

15) $\int \mathbf{sen}\sqrt{x}\, \boldsymbol{dx}$; $x = t^2$; $dx = 2tdt$; $t = \sqrt{x}$

$\int sen\sqrt{t^2} 2tdt = \int sent2tdt$ u = 2t , du = 2dt ; dv = sentdt , v = -cost

$\int sent2tdt = -2tcost + \int cost2dt = -2tcost + 2sent + C = 2\sqrt{x}cos\sqrt{x} + 2sen\sqrt{x} + C$

7. Halla el valor de las siguientes integrales, usando el método de integración por partes.

1) **∫3x cosx dx =** (dv = cos x dx u = 3x ; v = sen x du = 3 dx)

=3x · (sen x) - ∫ sen x · 3 dx = 3x · sen x + 3 · cos x + C

2) ∫x² · sen x dx = (dv = sen x dx u = x² ; v = -cos x du = 2x dx)

=x² · (-cos x) - ∫ (- cos x)· 2x dx = (dv = -cos x dx u = 2x ; v = - sen x du = 2 dx)

= (x² · (- cos x)) – [2x · (- sen x) - ∫ (- sen x) · 2 dx]=

= x² · (- cos x) + 2x · (sen x) - 2 · (-cos x) + C =

= - x² cos x + 2x sen x + 2 cos x + C

3) $\int x^2 \ln x\, dx =$ $\quad$ ($dv = x^2\,dx \quad u = \ln x \quad ; \quad v = \frac{x^3}{3} \quad du = \frac{1}{x}$)

$= \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}\,dx = \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3x}\,dx =$

$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3}\int x^2\,dx = \ln x \cdot \frac{x^3}{3} - \frac{x^3}{9} + C$

4) $\int \sqrt{x} \cdot \ln x\, dx =$ $\quad$ ($dv = \sqrt{x}\,dx = x^{\frac{1}{2}}\,dx \quad u = \ln x; \quad v = = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \quad du = \frac{1}{x}dx$)

$= \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \int \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \cdot \frac{1}{x}\,dx = \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \int \frac{2x^{\frac{1}{2}}}{3}\,dx =$

$= \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2}{3}\int x^{\frac{1}{2}}\,dx = \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4x^{\frac{3}{2}}}{9} + C$

5) $\int \frac{\ln x}{x^2}\,dx =$ $\quad$ ($dv = x^{-2}\,dx \quad u = \ln x \quad ; \quad v = \frac{x^{-1}}{-1} \quad du = \frac{1}{x}\,dx$)

$= \int \ln x \cdot x^{-2}\,dx = \ln x \cdot \frac{x^{-1}}{-1} - \int \frac{x^{-1}}{-1} \cdot \frac{1}{x}\,dx = \ln x \cdot \frac{x^{-1}}{-1} - \int \frac{x^{-1}}{-x}\,dx =$

$= \ln x \cdot \frac{x^{-1}}{-1} + \int \frac{1}{x^2}\,dx = -\ln x\ \frac{1}{x} + \int \frac{1}{x^2}dx = -\ln x\ \frac{1}{x} + \frac{x^{-1}}{-1} + C =$

$= -\ln x\ \frac{1}{x} - \frac{1}{x} + C$

6) $\int 2e^x \cdot \cos x\,dx =$

Cambios 1		Cambios 2	
$dv = \cos x\,dx$	$v = \text{sen } x$	$dv = \text{sen } x$	$v = -\cos x$
$u = 2e^x$	$du = 2e^x dx$	$u = 2e^x$	$du = 2e^x\,dx$

(cambios 1) $\quad = 2e^x \cdot \text{sen } x - \int \text{sen } x \cdot 2e^x dx =$

(cambios 2) $\quad = 2e^x \cdot \text{sen } x - [2e^x \cdot (-\cos x) - \int(-\cos x) \cdot 2e^x\,dx\,] =$

$= 2e^x \cdot \text{sen } x - [2e^x \cdot (-\cos x) + \int \cos x \cdot 2e^x\,dx\,] =$

$= 2e^x \cdot \text{sen } x + 2e^x \cdot \cos x - \int \cos x \cdot 2e^x\,dx\,]$ $\quad$ (haciendo $\int \cos x \cdot 2e^x dx = I$)

$I = 2e^x \cdot \text{sen } x + 2e^x \cdot \cos x - I$

$2I = 2e^x \cdot \text{sen } x + 2e^x \cdot \cos x$

$I = e^x \cdot \text{sen } x - e^x \cdot \cos x + C$

7) $\int 2e^x \cdot \text{senx}\,dx$

- Fórmula: $\quad \int u \cdot dv = u \cdot v - \int v \cdot du$
- $u = 2e^x \to du = 2e^x dx$
- $dv = senxdx \to v = \int senx\,dx = -cosx$

$\int 2e^x \cdot \text{senx}\,dx = 2e^x \cdot (-\text{cosx}) - \int -\cos x \cdot 2e^x dx = 2e^x \cdot (-\text{cosx}) + \int \cos x \cdot 2e^x dx =$

Repetimos el método de integración por partes:

- $u = 2e^x \rightarrow du = 2e^x dx$
- $dv = cosxdx \rightarrow v = \int cosx\, dx = senx$

$= 2e^x \cdot (-cosx) + 2e^x \cdot senx - \int senx \cdot 2e^x dx$

Hacemos $\int \mathrm{senx} \cdot 2\mathrm{e^x} = \mathrm{I}$ de donde,

$I = 2e^x \cdot (-cosx) + 2e^x \cdot senx - I$

$2I = 2e^x \cdot (-cosx) + 2e^x \cdot senx$

$I = e^x \cdot (-cosx) + e^x \cdot senx$

$I = e^x \cdot (senx - cosx) + C$

8) $\int \boldsymbol{e^x \cdot cos3x\, dx}$

- $u = e^x \rightarrow du = e^x dx$
- $dv = cos3xdx \rightarrow v = \int cos3x\, dx = \frac{1}{3} \cdot sen3x$

$\int \mathrm{e^x} \cdot \cos 3\mathrm{x}\, \mathrm{dx} = \frac{\mathrm{e^x}}{3}\mathrm{sen3x} - \int \frac{\mathrm{e^x}}{3}\, \mathrm{sen3x\, dx} = \frac{\mathrm{e^x}}{3}\mathrm{sen3x} - \frac{1}{3}\int \mathrm{e^x}\ \mathrm{sen3x\, dx} =$

Repetimos el método de integración por partes:

- $u = e^x \rightarrow du = e^x dx$
- $dv = sen3xdx \rightarrow v = \int sen3x\, dx = -\frac{1}{3}cos3x$

$= \frac{e^x}{3}sen3x - \frac{1}{3}(-\frac{e^x}{3} \cdot cos3x - \frac{1}{3} \cdot \int e^x(-cos3x)\, dx)$

Hacemos $\int \mathrm{e^x cos3x\, dx} = \mathrm{I}$, de donde,

$I = \frac{e^x}{3}sen3x + \frac{e^x}{9}cos3x - \frac{1}{9} \cdot I$

$\frac{10}{9} \cdot I = \frac{e^x}{3} \cdot sen3x + \frac{e^x}{9}cos3x$

$I = \frac{e^x}{10} \cdot \left(3sen3x + \frac{cos3x}{3}\right) + C$

9) $\int \boldsymbol{\frac{4-2\,x^2}{x} \cdot ln(x)dx}$

- $u = \ln(x) \rightarrow du = \frac{1}{x}dx$
- $\mathrm{dv} = \frac{4-2\,\mathrm{x}^2}{\mathrm{x}}\mathrm{dx} \rightarrow \mathrm{v} = \int \left(\frac{4}{\mathrm{x}} - 2\mathrm{x}\right)\mathrm{dx} = 4 \cdot \ln(\mathrm{x}) - \mathrm{x}^2$

$\int \frac{4-2\,x^2}{x} \cdot ln(x)\, dx = \ln(x) \cdot (4 \cdot \ln(x) - x^2) - \int \frac{1}{x} \cdot (4\ln(x) - x^2)dx =$

$= \ln(x) \cdot (4 \cdot \ln(x) - x^2) - \int \frac{4}{x} \cdot \ln(x)\, dx + \int xdx =$

$= \ln(x) \cdot (4 \cdot \ln(x) - x^2) + \frac{x^2}{2} - 4\int \frac{\ln(x)}{x}dx =$ $\quad \left[4\int \ln(x) \cdot \frac{1}{x}dx = 4\frac{ln^2(x)}{2}\right]$

$$= 4ln^2(x) - x^2\ln(x) + \frac{x^2}{2} - \frac{4ln^2(x)}{2} = 2ln^2(x) - x^2\ln(x) + \frac{x^2}{2} + C$$

8. Halla el valor de las siguientes integrales racionales:

1) $\int \frac{2}{x^2+1}dx$

Las raíces del denominador son raíces complejas.

$x^2 + 1 = 0; \quad x = \pm i$ luego:

$$\int \frac{2}{x^2+1}dx = 2\int \frac{1}{x^2+1}dx = 2arctan(x) + C$$

2) $\int \frac{3}{2x^2+2}dx$

Las raíces del denominador son raíces complejas.

$2x^2 + 2 = 0; \quad x = \pm i$ luego:

$$\int \frac{3}{2x^2+2}\mathrm{dx} = \int \frac{3}{2(x^2+1)}\mathrm{dx} = \frac{3}{2}\int \frac{1}{x^2+1}\mathrm{dx} = \frac{3}{2}\mathrm{arctan}(x) + C = \frac{3\mathrm{arctan}\,(x)}{2} + C$$

3) $\frac{3}{x-3}dx$

La raíz del denominador es una raíz real simple.

$x - 3 = 0; \quad x = 3$ luego:

$$\int \frac{3}{x-3}dx = 3\int \frac{1}{x-3}dx = 3ln|x-3| + C$$

4) $\int \frac{2}{3x^2+3}dx$

$$\int \frac{2}{3x^2+3}dx = \int \frac{2}{3(x^2+1)}dx = \frac{2}{3}\int \frac{1}{x^2+1}dx = \frac{2}{3}\arctan(x) + C = \frac{2\arctan\,(x)}{3} + C$$

5) $\int \frac{5x}{x^2+3}dx$

$$\int \frac{5x}{x^2+3}dx = \frac{5}{3}\int \frac{3x}{(x^2+3)}dx = \frac{5}{3}\ln|x^2+3| + C$$

6) $\int \frac{3x-2}{x^2+1}dx$

$$\int \frac{3x-2}{x^2+1}dx = \int \left(\frac{3x}{x^2+1} - \frac{2}{x^2+1}\right)dx = \frac{3}{2}\int \frac{2x}{x^2+1}dx - 2\int \frac{1}{x^2+1}dx = \frac{3\ln|x^2+1|}{2} - 2\arctan(x) + C$$

7. $\int \frac{(2x-3)^2}{3x^2}\mathrm{dx}$

IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

$$\int \frac{(2x-3)^2}{3x^2}dx = \int \frac{4x^2+9-12x}{3x^2}dx = \int \frac{4}{3}dx + \int \frac{-12x}{3x^2}dx + \int \frac{9}{3x^2}dx =$$

$$= \frac{4}{3}\int dx - 4\int \frac{1}{x}dx + 3\int \frac{1}{x^2}dx = \frac{4}{3}x - 4\ln|x| + 3\int x^{-2}dx =$$

$$= \frac{4}{3}x - 4\ln|x| + 3\frac{x^{-1}}{-1} + C = \frac{4}{3}x - 4\ln|x| - \frac{3}{x} + C$$

8. $\int \frac{x+2}{x+1}dx$

Según la división euclídea, obtenemos:

C(x)=1 R(x)=1 Q(x)=x+1

$$\int \frac{x+2}{x+1}dx = \int \left(1 + \frac{1}{x+1}\right)dx = \int 1dx + \int \frac{1}{x+1}dx = x + \ln|x+1| + C$$

9. $\int \frac{x-1}{x+1}dx$

Según la división euclídea, obtenemos:

C(x)=1 R(x)=-2 Q(x)=x+1

$$\int \frac{x-1}{x+1}dx = \int (1 + \frac{-2}{x+1})dx = \int 1dx + \int \frac{-2}{x+1}dx = x - 2\ln|x+1| + C$$

10) $\int \frac{3x-1}{x+3}dx$;

Efectuando la división obtenemos: C(x): 3 R(x): -10 Q(x): $x+3$

$$\int \frac{P(x)}{Q(x)}\,dx = \int C(x)\,dx + \int \frac{R(x)}{Q(x)}\,dx$$

$$\int \frac{3x-1}{x+3}dx = \int 3\,dx + (-10)\int \frac{1}{x+3}dx;\ 3x - 10\ln(|x+3|) + C$$

11) $\int \frac{3x^3}{x^2-4}dx$; C(x): $3x$ R(x): $12x$ Q(x): x^2-4

$$\int \frac{3x^3}{x^2-4}dx = \int 3x\,dx + 6\int \frac{\frac{12x}{6}}{x^2-4}dx;\ \frac{3x^2}{2} + 6\ln(|x^2-4|) + C$$

12) $\int \frac{3x^3}{x^2-1}dx$; C(x): $3x$ R(x): $3x$ Q(x): x^2-1

$$\int \frac{3x^3}{x^2-1}dx = \int 3x\,dx + \int \frac{3x}{x^2-1}dx;\ \int 3x\,dx + \frac{3}{2}\int \frac{2x}{(x^2-1)}dx;\ \frac{3x^2}{2} + \frac{3}{2}\ln(|x^2-1|) + C$$

13) $\int \frac{x^2+2x+2}{x+2}dx$; efectuando la división obtenemos C(x) = x; R(x) = 2; de donde

IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

$\int \frac{x^2+2x+2}{x+2}dx = \int (x)dx + \int \frac{2}{x+2}dx = \int (x)dx + 2\int \frac{1}{x+2}dx = \frac{x^2}{2} + 2\ln|x+2| + C$

14) $\int \frac{x^3+4x^2-2x+5}{x-2}dx$; efectuando la división, C(x) = $x^2 + 6x + 10$; R(x) = -25; de donde

$\int \frac{x^3+4x^2-2x+5}{x-2}dx = \int (x^2+6x+10)dx + \int \frac{-25}{x-2}dx =$

$= \int (x^2+6x+10)dx - 25\int \frac{1}{x-2}dx = \frac{x^3}{3} + 3x^2 + 10x - 25\ln|x-2| + C$

15) $\int \frac{2}{x^2-4}dx$; hacemos la descomposición en fracciones simples,

$\frac{2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$ 2 = A(x+2) + B(x-2), sustituyendo por x = y x = -2 , $A = \frac{1}{2}$, $B = -\frac{1}{2}$

$\int \frac{2}{(x-2)(x+2)}dx = \int \frac{A}{x-2}dx + \int \frac{B}{x+2}dx = \int \frac{1}{2(x-2)}dx + \int -\frac{1}{2(x+2)}dx = \int \frac{1}{2x-2}dx - \int \frac{1}{2x+2}dx =$
$\frac{1}{2}\int \frac{2}{2x-2}dx - \frac{1}{2}\int \frac{2}{2x+2}dx = \frac{1}{2}\ln|x-2| - \frac{1}{2}\ln|x+2| + C \quad = \frac{1}{2}\ln\left|\frac{x-2}{x+2}\right| + C$

16) $\int \frac{3x+2}{x^2+3x}dx$

1. Se analizan las raíces del denominador: $x^2 + 3x = 0; x(x+3) = 0 \rightarrow x = 0; x = -3$

2. Se escribe la descomposición en fracciones simples:
$$\frac{3x+2}{x^2+3x} = \frac{A}{x} + \frac{B}{(x+3)}$$

3. Para hallar A y B, se sustituye la x por el valor de las raíces:

$3x + 2 = A(x+3) + Bx$

$x = 0 \rightarrow 2 = 3A \rightarrow A = \frac{2}{3}$

$x = -3 \rightarrow -7 = -3B \rightarrow B = \frac{7}{3}$

4. Se escribe la integral como suma de integrales:

$$\int \frac{3x+2}{x^2+3x}dx = \int \frac{2/3}{x}dx + \int \frac{7/3}{(x+3)}dx = \frac{2}{3}\ln|x| + \frac{7}{3}\ln|x+3| + C$$

17) $\int \frac{4x+3}{x^2-1}dx$

1. Se analizan las raíces del denominador: $x^2 - 1 = 0; x = \sqrt{1} \rightarrow x = 1; x = -1$

2. Se escribe la descomposición en fracciones simples:
$$\frac{4x+3}{x^2-1} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$$

3. Para hallar A y B, se sustituye la x por el valor de las raíces:

$$4x+3=A(x+1)+B(x-1)$$

$$x=1\rightarrow\ 7=2A\rightarrow A=\frac{7}{2}$$
$$x=-1\rightarrow -1=-2B\rightarrow B=\frac{1}{2}$$

4. Se escribe la integral como suma de integrales:

$$\int\frac{4x+3}{x^2-1}\,dx=\int\frac{7/2}{(x-1)}dx+\int\frac{1/2}{(x+1)}dx=\frac{7}{2}\ln|x-1|+\frac{1}{2}\ln|x+1|+C$$

18) $\int\frac{3x^2}{x^2+6x+9}dx$

1. Al ser el grado del polinomio del numerador igual que el del denominador, debemos realizar la división de estos, $3x^2:(x^2+6x+9)\rightarrow$ y obtenemos como cociente 3 y como resto $-18x-27$
   a. Obtenemos la siguiente integral: $\int 3dx+\int\frac{-18x-27}{x^2+6x+9}dx\rightarrow 3x+\int\frac{-18x-27}{x^2+6x+9}dx$
2. Con la nueva integral obtenida, realizamos el procedimiento habitual para integrales racionales:
   a. Igualamos el polinomio del divisor a cero: $x^2+6x+9=0\rightarrow x=-3$ (doble)
   b. Con las soluciones obtenidas calculamos: $\frac{-18x-27}{x^2+6x+9}=\frac{A}{x+3}+\frac{B}{(x+3)^2}\rightarrow$

   $$-18x-27=A(x+3)^2+B(x+3)$$

   Sustituimos x por un valor cualquiera:

   $x=-2\rightarrow 9=A+B\,;\quad x=-1\rightarrow -9=4A+4B$

   Resolvemos el sistema de ecuaciones obtenido: $A=-\frac{27}{2};\quad B=\frac{45}{2}$

3. Gracias a este procedimiento hemos obtenido:

$$3x+\int\frac{\frac{-27}{2}}{x+3}\,dx+\int\frac{\frac{45}{2}}{(x+3)^2}dx=3x-\frac{27}{2}\int\frac{dx}{x+3}+\frac{45}{2}\int\frac{dx}{(x+3)^2}=$$

$$3x-\frac{27}{2}ln|x+3|+\frac{45}{2}\cdot\frac{(x-3)^{-1}}{-1}=3x-\frac{27}{2}ln|x+3|-\frac{45}{2}\cdot\frac{1}{x+3}+C$$

IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

19) $\int \frac{x+2}{x^2-5x+6}dx$

1. Como el grado del polinomio del denominador es superior al del polinomio del numerador, utilizamos el procedimiento habitual para estos casos:
   a. Igualamos el polinomio del denominador a cero: $x^2-5x+6=0 \to x_1=3; x_2=2$
   b. Con las soluciones obtenidas calculamos:

   $\frac{x+2}{x^2-5x+6}=\frac{A}{x-3}+\frac{B}{x-2} \to x+2=A(x-2)+b(x-3)$

   Sustituimos x por un valor cualquiera:

   $x=2 \to 4=-B \to B=-4;$ $x=3 \to 5=A$

2. Gracias a este procedimiento obtenemos:

   $\int \frac{5}{x-3}dx+\int \frac{-4}{x-2}dx=5ln|x-3|-4ln|x-2|+C$

20) $\int \frac{3x-2}{x^2-4x+4}dx$

1. Como el grado del polinomio del denominador es superior al del polinomio del numerador, utilizamos el procedimiento habitual para estos casos:
   a. Igualamos el polinomio del divisor a cero: $x^2-4x+4=0 \to x=2$ (doble)
   b. Con las soluciones obtenidas calculamos:

   $\frac{3x-2}{x^2-4x+4}=\frac{A}{x-2}+\frac{B}{(x-2)^2} \to 3x-2=A(x-2)^2+B(x-2)$

   Sustituimos x por un valor cualquiera: $x=3 \to 7=A+B; x=1 \to 1=A-B$

   Resolvemos el sistema de ecuaciones obtenido: $A=4; B=3$

2. Gracias a este procedimiento obtenemos:

   $\int \frac{4}{x-2}dx+\int \frac{3}{(x-2)^2}dx=4ln|x-2|+3\cdot\frac{(x-2)^{-1}}{-1}=4ln|x-2|-\frac{3}{x-2}+C$

21) $\int \frac{3x+1}{x^3-4x^2+3x}dx$

$\frac{3x+1}{x^3-4x^2+3x}=\frac{3x+1}{x(x-3)(x-1)}=\frac{A}{x}+\frac{B}{x-3}+\frac{C}{x-1} \Longrightarrow \frac{3x+1}{x(x-3)(x-1)}=\frac{A(x-3)(x-1)+Bx(x-1)+Cx(x-3)}{x(x-3)(x-1)}$

$3x+1=(Ax-3A)(x-1)+Bx^2-Bx+Cx^2-3Cx$

$3x+1=Ax^2-Ax-3Ax+3A+Bx^2-Bx+Cx^2-3Cx$

$3x+1=(A+B+C)x^2-(4A+B+3C)x+3A$

$A + B + C = 0$

$-(4A + B + 3C) = 3$

$3A = 1 \Rightarrow \mathbf{A = \frac{1}{3}}$

$$\begin{cases} \frac{1}{3} + B + C = 0 \\ \frac{4}{3} + B + 3C = 3 \end{cases} \begin{cases} B + C = -\frac{1}{3} \\ B + 3C = 3 - \frac{4}{3} \end{cases} \begin{cases} B + C = -\frac{1}{3} \\ -B - 3C = 3 + \frac{4}{3} \end{cases} \{-2C = 4 \Rightarrow C = \frac{4}{-2} \Rightarrow \mathbf{C = -2}$$

$B = -A - C = -\frac{1}{3} - (-2) \Rightarrow \mathbf{B = \frac{5}{3}}$

$\int \frac{3x+1}{x^3-4x^2+3x} dx = \int \frac{\frac{1}{3}}{x} dx + \int \frac{\frac{5}{3}}{x-3} dx + \int \frac{-2}{x-1} dx = \frac{1}{3} \int \frac{dx}{x} + \frac{5}{3} \int \frac{dx}{x-3} - 2 \int \frac{dx}{x-1} =$

$= \mathbf{\frac{1}{3} \ln(x) + \frac{5}{3} \ln(x-3) - 2\ln(x-1) + C}$

22) $\int \frac{2x^2-1}{x^2+3x+2} dx$ efectuando la división obtenemos, C(x) = 2; R(x) = - 6x – 5 , de donde,

$\frac{D}{d} = C + \frac{R}{d} \Rightarrow \int \frac{2x^2-1}{x^2+3x+2} dx = \int 2dx + \int \frac{-(6x+5)}{x^2+3x+2} dx = 2 \int dx - \int \frac{6x+5}{x^2+3x+2} dx = I_1 + I_2$

$I_1 = 2 \int dx = 2x + C$

$I_2 = \int \frac{6x+5}{x^2+3x+2} \Rightarrow \frac{6x+5}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$

$\frac{6x+5}{(X+2)(x+1)} = \frac{A(x+1)+B(x+2)}{(x+2)(x+1)} \Rightarrow 6x + 5 = Ax + A + Bx + 2B$

$6x + 5 = (A + B)x + A + 2B$

$A + B = 6$

$A + 2B = 5$

$B = -1$

$A = 7$

$\int \frac{7}{x+2} dx + \int \frac{-1}{x+1} dx = 7 \int \frac{dx}{x+2} - \int \frac{dx}{x+1} = 7\ln(x+2) - \ln(x+1) + C$

$\int \frac{2x^2-1}{x^2+3x+2} dx = 2x - 7\ln(x+2) - \ln(x+1) + C$

23) $\int \frac{x-1}{x^2-4x+4} dx$

$\frac{x-1}{x^2-4x+4} = \frac{x-1}{(x-2)^2} = \frac{A}{(x-2)^2} + \frac{B}{(x-2)} \Rightarrow \frac{x-1}{(x-2)^2} = \frac{A+B(x-2)}{(x-2)^2} \Rightarrow x - 1 = A + Bx - 2B$

$B = 1$

$A - 2B = -1 \Rightarrow A - 2(1) = -1 \Rightarrow A = 1$

$\int \frac{x-1}{x^2-4x+4} dx = \int \frac{1}{(x-2)^2} dx + \int \frac{1}{x-2} dx = I_1 + I_2$

$I_1 \Longrightarrow \int \frac{dx}{(x-2)^2}\,dx \begin{cases} z = x-2 \\ dz = dx \end{cases} = \int \frac{dz}{z^2} = \int z^{-2}dz = \frac{z^{-1}}{-1} + C = -\frac{1}{z} + C = -\frac{1}{x-2} + C$

$I_2 \Longrightarrow \int \frac{dx}{x-2} \begin{cases} z = x-2 \\ dz = dx \end{cases} = \int \frac{dz}{z} = \ln(z) + C = \ln|x-2| + C$

$\int \frac{x-1}{x^2-4x+4}dx = -\frac{1}{x-2} + \ln(x-2) + C$

24) $\int \frac{3x-1}{x^2+6x+9}dx = \int \frac{3x-1}{(x+3)(x+3)}dx = \int \frac{3x-1}{(x+3)^2}dx$

$\frac{3x-1}{(x+3)^2} = \frac{A}{(x+3)^2} + \frac{B}{(x+3)} \;\rightarrow\; \frac{3x-1}{(x+3)^2} = \frac{A+B(x+3)}{(x+3)^2}$

$3x-1 = A + B(x+3)$

$x = -3;\; 3\cdot(-3) - 1 = A + B(-3+3) \;\rightarrow\; A = 10$

$x = 3;\; 3\cdot 3 - 1 = -10 + B(3+3) \rightarrow 8 = -10 + 6B \rightarrow B = \frac{18}{6} = 3$

$\int \frac{3x-1}{x^2+6x+9}dx = \int \frac{-10}{(x+3)^2}dx + \int \frac{3}{(x+3)}dx = -10\int \frac{1}{(x+3)^2}dx + 3\int \frac{1}{(x+3)}dx =$

$= -10\int (x+3)^{-2}\,dx + 3\int \frac{1}{(x+3)}dx = -10\cdot\frac{(x+3)^{-1}}{-1} + 3\ln|x+3| + C = \frac{10}{(x+3)} + 3\ln|x+3| + C$

9. Halla el valor de las siguientes integrales definidas

1) $\int_1^3 \frac{dx}{2x} = \int_1^3 \frac{1}{2x}dx = \frac{1}{2}\int_1^3 \frac{1}{x}dx = \frac{1}{2}\ln(|x|)\Big|_1^3 = \frac{1}{2}\ln(|3|) - \frac{1}{2}\ln(|1|) = \frac{1}{2}\ln(3)$

2) $\int_2^3 \frac{x}{x^2-1}dx = \int_2^3 \frac{1}{2t}dt = \frac{1}{2}\int_2^3 \frac{1}{t}dt = \frac{1}{2}\ln(|t|)\Big|_2^3 = \frac{1}{2}\ln(|x^2-1|)\Big|_2^3 = \frac{1}{2}\ln(|3^2-1|) - \frac{1}{2}\ln(|2^2-1|) = \frac{1}{2}\ln\left(\frac{8}{3}\right)$

3) $\int_{\frac{\pi}{4}}^{\frac{5\pi}{3}} \sin(x)\,dx = -\cos(x)\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{3}} = -\cos\left(\frac{5\pi}{3}\right) - \left(-\cos\left(\frac{\pi}{4}\right)\right) = \frac{-1+\sqrt{2}}{2}$

4) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin(3x)\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin(t)}{3}dt = \frac{1}{3}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin(t)\,dt = \frac{1}{3}(-\cos(t))\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{1}{3}(-\cos(3x))\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = -\frac{\cos(3x)}{3}\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = -\frac{\cos\left(3\times\frac{\pi}{4}\right)}{3} - \left(-\frac{\cos\left(3\times\frac{\pi}{6}\right)}{3}\right) = \frac{\sqrt{2}}{6}$

5) $\int_{-4}^4 |x|\,dx = \int_{-4}^0 (-x)\,dx + \int_0^4 x\,dx = -\frac{x^2}{2}\Big|_{-4}^0 + \frac{x^2}{2}\Big|_0^4 = 8 + 8 = 16$

6) $\int_{-1}^1 \left(3x^2 - 2x + \frac{1}{2}\right)dx = \int_{-1}^1 3x^2\,dx - \int_{-1}^1 2x\,dx + \int_{-1}^1 \frac{1}{2}dx = \left(x^3 - x^2 + \frac{1}{2}x\right)\Big|_{-1}^1 = \left(1^3 - 1^2 + \frac{1}{2}\cdot 1\right) - \left((-1)^3 - (-1)^2 + \frac{1}{2}\cdot(-1)\right) = 3$

7) $\int_{-1}^{2}\left(\frac{2}{x+2}-\frac{3}{x-3}\right)dx = \int_{-1}^{2}\frac{2}{x+2}dx - \int_{-1}^{2}\frac{3}{x-3}dx = (2\ln(|x+2|)-3\ln(|x-3|))|_{-1}^{2} = 10\ln(2)$

8) $\int_{-2}^{2}\left(\frac{3a}{5}-\frac{x}{2}\right)dx = \int_{-2}^{2}\frac{3a}{5}dx - \int_{-2}^{2}\frac{x}{2}dx = \left(\frac{3ax}{5}-\frac{x^2}{4}\right)\Big|_{-2}^{2} = \frac{3a\cdot 2}{5}-\frac{2^2}{4}-\left(\frac{3a\cdot(-2)}{5}-\frac{(-2)^2}{4}\right) = \frac{12}{5}a$

9) $\int_{2}^{3}\frac{1}{x\cdot(\ln x)^3}dx$

$\int\frac{1}{x\cdot(\ln x)^3}dx = \int(\ln x)^{-3}\cdot\frac{1}{x}dx = \frac{(\ln x)^{-2}}{-2} = -\frac{1}{2\cdot(\ln x)^2}+C$

$\int_{2}^{3}\frac{1}{x\cdot(\ln x)^3}dx = -\frac{1}{2\cdot(\ln x)^2}\Big|_2^3 = -\frac{1}{2\cdot(\ln 3)^2}+\frac{1}{2\cdot(\ln 2)^2} = -0{,}414+1{,}04 = 0{,}626$

10) $\int_{-2}^{0}\left(e^{2x}+\frac{3}{e^{3x}}\right)dx$

$\int\left(e^{2x}+\frac{3}{e^{3x}}\right)dx = \int e^{2x}dx+\int e^{-3x}\cdot 3dx = \frac{1}{2}\int e^{2x}\cdot 2\,dx - \int e^{-3x}\cdot(-3)\,dx$

$= \frac{1}{2}e^{2x}-e^{-3x}+C$

$\int_{-2}^{0}\left(e^{2x}+\frac{3}{e^{3x}}\right)dx = \frac{1}{2}e^{2x}-e^{-3x}\Big|_{-2}^{0} = \left(\frac{1}{2}e^{2\cdot 0}-e^{-3\cdot 0}\right)-\left(\frac{1}{2}e^{2\cdot(-2)}-e^{-3\cdot(-2)}\right) = 403{,}91$

11) $\int_{\frac{\pi}{4}}^{\frac{5\pi}{3}}(\operatorname{sen}x-\cos x)^2\,dx$

$\int_{\frac{\pi}{4}}^{\frac{5\pi}{3}}(\operatorname{sen}x-\cos x)^2\,dx = \int_{\frac{\pi}{4}}^{\frac{5\pi}{3}}(\operatorname{sen}^2 x-2\operatorname{sen}x\cos x+\cos^2 x)dx =$

$= \int_{\frac{\pi}{4}}^{\frac{5\pi}{3}}(1-\cos^2 x-2\operatorname{sen}x\cos x+\cos^2 x)dx = (x-\operatorname{sen}^2 x)|_{\frac{\pi}{4}}^{\frac{5\pi}{3}} =$

$= \left[\frac{5\pi}{3}-\operatorname{sen}^2\frac{5\pi}{3}\right]-\left[\frac{\pi}{4}-\operatorname{sen}^2\frac{\pi}{4}\right] = [5{,}23]-[0{,}79] = 4{,}4\text{u}^2$

10. Halla el valor de b para que se cumpla $\int_{-1}^{b}(2bx-3x^2)dx=-12$.

1. Se resuelve la integral con la incógnita b:

$$\int_{-1}^{b}(2bx-3x^2)dx = 2b\frac{x^2}{2}-3\frac{x^3}{3}\Big]_{-1}^{b} = bx^2-x^3]_{-1}^{b}$$

2. Sustituimos los límites de integración:

$$(b\cdot b^2-b^3)-(b\cdot(-1)^2-(-1)^3) = b^3-b^3-b-1 = -b-1$$

3. Igualamos el resultado a -12:

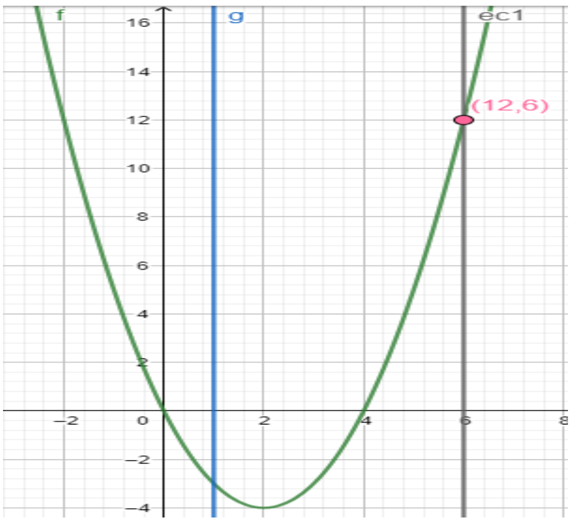
IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

$$-b - 1 = -12 \rightarrow -b = -12 + 1 \rightarrow -b = -11 \rightarrow b = 11$$

Resultado: b=11

11. Halla el área entre la función $f(x) = x^2 - 4x$, el eje de abscisas, y las rectas x=1 y x=6.

1. Hacemos el gráfico:



2. Hallamos los cortes con el eje x de la función:

$$f(x) = x^2 - 4x \rightarrow x^2 - 4x = 0 \rightarrow x(x-4) = 0 \rightarrow x_1 = 0, \qquad x_2 = 4$$

3. Hallamos el área de las dos zonas de áreas obtenidas; de x=1 a x=4, y de x=4 a x=6:

$$\int_1^4 (x^2 - 4x)dx = \left.\frac{x^3}{3} - 4\frac{x^2}{2}\right]_1^4 = \left.\frac{x^3}{3} - 2x^2\right]_1^4 = \left(\frac{4^3}{3} - 2\cdot 4^2\right) - \left(\frac{1^3}{3} - 2\cdot 1^2\right) = -9$$

Como es un área tomamos su valor positivo, 9.

$$\int_4^6 (x^2 - 4x)dx = \left.\frac{x^3}{3} - 4\frac{x^2}{2}\right]_4^6 = \left.\frac{x^3}{3} - 2x^2\right]_4^6 = \left(\frac{6^3}{3} - 2\cdot 6^2\right) - \left(\frac{4^3}{3} - 2\cdot 4^2\right) = \frac{32}{3}$$

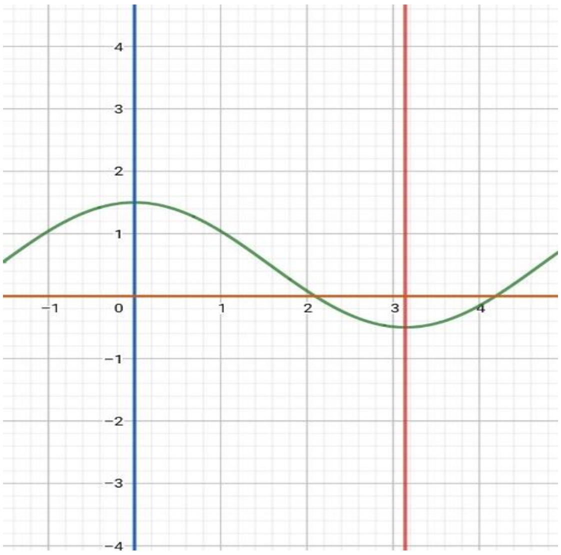
4. Sumamos ambas áreas:

$$\frac{32}{3} + (9) = \frac{59}{3}\ u.a.$$

Resultado: El área es $\frac{59}{3}$ u.a.

IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

12. Halla el área limitada por la función f(x) = 0,5 + cosx, el eje de abcisas y las rectas x = 0 y x = π.



a=0

b=π

c = $0,5+\cos x=0$; $\cos x=-0,5$

$x=arc\cos(-0,5)=\frac{2\pi}{3}$

At=A1+A2;

A1= $\int_a^c f(x)dx=\int_0^{\frac{2\pi}{3}}(0,5+\cos x)dx=\int_0^{\frac{2\pi}{3}}0,5dx+\int_0^{\frac{2\pi}{3}}\cos xdx=(0,5x+\sin x)\begin{cases}\frac{2\pi}{3}\\0\end{cases}=$

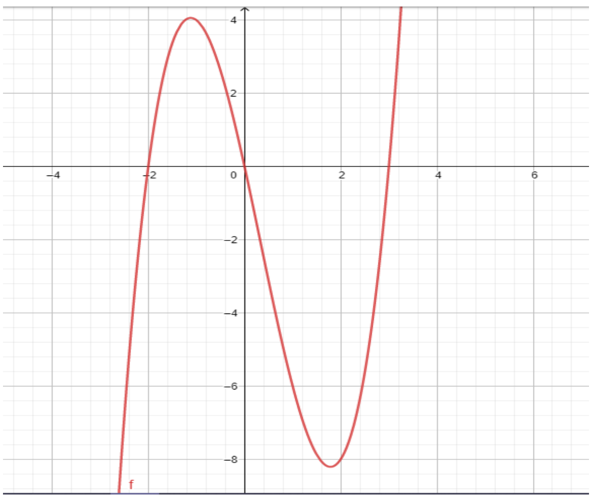
$=\left[0,5\left(\frac{2\pi}{3}\right)+\sin\left(\frac{2\pi}{3}\right)\right]-[0,5(0)+\sin(0)]=1,08-0=1,08u^2$

A2=$\left|\int_c^b f(x)dx\right|=\left|\int_{\frac{2\pi}{3}}^{\pi}(0,5+\cos x)dx\right|=$ $\left|(0,5x+\text{sen}\,x)\begin{cases}\pi\\\frac{2\pi}{3}\end{cases}\right|=\left|[0,5(\pi)+\text{sen}(\pi)]-\left[0,5\left(\frac{2\pi}{3}\right)+\sin(\frac{2\pi}{3})\right]\right|=0,55$

At= 1,08+0,55= $1,63u^2$

13. Halla el área de la región limitada por la función $f(x)=x^3-x^2-6x$ y el eje de abscisas.

1. Hacemos el gráfico:



IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

2. Hallamos los cortes con el eje x de la función:
$f(x) = x^3 - x^2 - 6x \rightarrow x^3 - x^2 - 6x = 0 \rightarrow x(x^2 - x - 6) = 0 \rightarrow \quad x_1 = 0, x_2 = -2, x_3 = 3$

3. Hallamos el área de las dos zonas obtenidas; de x=-2 a x=0, y de x=0 a x=3:

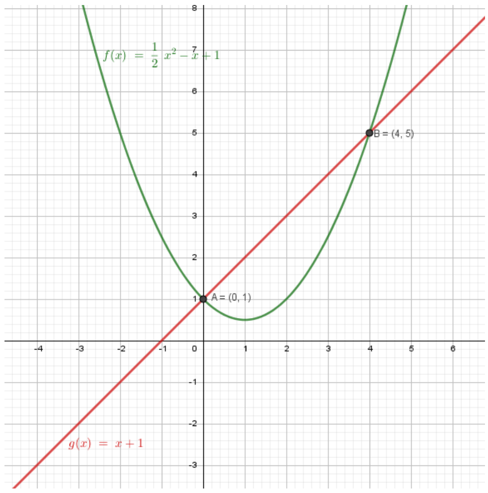
$$\int_{-2}^{0} (x^3 - x^2 - 6x)dx - \int_{0}^{3} (x^3 - x^2 - 6x)dx =$$

$$\left.\frac{x^4}{4} - \frac{x^3}{3} - 3x^2\right]_{-2}^{0} - \left.\left(\frac{x^4}{4} - \frac{x^3}{3} - 3x^2\right)\right]_{0}^{3} =$$

$$\left(\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - 3 \cdot (-2)^2\right) - \left(\frac{3^4}{4} - \frac{3^3}{3} - 3 \cdot 3^2\right) = 10{,}42u^2$$

Resultado: El área es $\frac{125}{12}$ u.a.

14. Calcula el área de la porción del plano que limitan las curvas $y = \frac{1}{2}x^2 - x + 1$ e y – x – 1 = 0



$$\text{Área} = \int_0^4 \left[(x+1) - \left(\frac{1}{2}x^2 - x + 1\right)\right] dx = \int_0^4 \left(\frac{1}{2}x^2 + 2x\right) dx = \left.\left(\frac{1}{2} \cdot \frac{x^3}{3} + x^2\right)\right]_0^4 = \frac{80}{3} u.a.$$

15. Halla el área delimitada por las gráficas:

a) $(x) = \sqrt{x}$ y $g(x) = x^2$

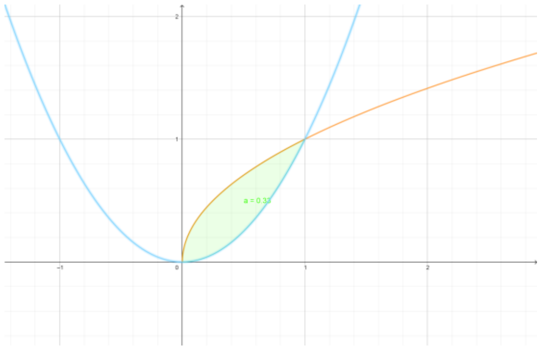
Igualamos f(x) y g(x) para hallar los puntos de corte:

$\sqrt{x} = x^2$; $(\sqrt{x})^2 = (x^2)^2$; $x = x^4$; $x^4 - x = 0$; $x(x^3 - 1) = 0$

$x = 0$

$(x^3 - 1) = 0$; $x^3 = 1$; $x = \sqrt[3]{1}$; $x = 1$

IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra



Los puntos de corte son x=0 y x=1.

Área=$\int_0^1 (f(x)-g(x))\,dx = \int_0^1 (\sqrt{x}-x^2)\,dx = \int_0^1 (x^{1/2}-x^2)\,dx = \frac{2\sqrt{x^3}}{3}-\frac{x^3}{3}\Big]_0^1 = \frac{2\sqrt{x^3}-x^3}{3}\Big]_0^1 = \left(\frac{2\sqrt{1^3}-1^3}{3}\right)-\left(\frac{2\sqrt{0^3}-0^3}{3}\right) = \left(\frac{1}{3}\right)-(0) = \frac{1}{3}u.a.$

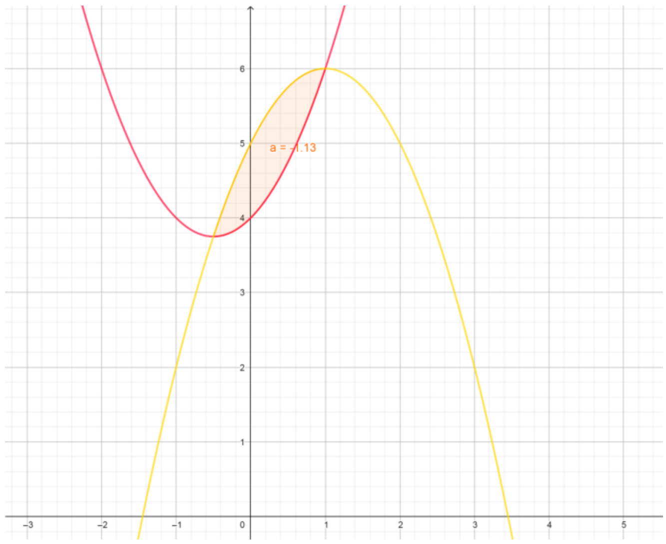
b) $\mathbf{f(x) = x^2 + x + 4 \quad y \quad g(x) = -x^2 + 2x + 5}$

Igualamos f(x) y g(x) para hallar los puntos de corte:

$x^2+x+4=-x^2+2x+5 \quad ; \quad 2x^2-x-1=0 \quad ; \quad x=1 \quad ; \quad x=-\frac{1}{2}$

Los puntos de corte son x=1 y x=$-\frac{1}{2}$

Área=$\left|\int_{-\frac{1}{2}}^{1}(f(x)-g(x))dx\right|$=$\left|\int_{-\frac{1}{2}}^{1}((x^2+x+4)-(-x^2+2x+5))\,dx\right|$=$\left|\int_{-\frac{1}{2}}^{1}(x^2+x+4+x^2-2x-5)\,dx\right| = \left|\int_{-\frac{1}{2}}^{1}(2x^2-x-1)\,dx\right| = \left|\frac{2x^3}{3}-\frac{x^2}{2}-x\Big]_{-\frac{1}{2}}^{1}\right| = \left|\left(\frac{2\cdot1^3}{3}-\frac{1^2}{2}-1\right)-\left(\frac{2\cdot\left(-\frac{1}{2}\right)^3}{3}-\frac{\left(-\frac{1}{2}\right)^2}{2}-\left(-\frac{1}{2}\right)\right)\right| = \left|\left(-\frac{5}{6}\right)-\left(\frac{7}{24}\right)\right| = \left|-\frac{5}{6}-\frac{7}{24}\right| = \left|-\frac{9}{8}\right| = \frac{9}{8}\,u.a. = 1{,}125\,u.a.$



IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

Ejercicios Autoevaluación

1) Los valores de a, b y c para los que $F(x) = ax^3 - be^x + c\sin x$ es una primitiva de la función $f(x) = 3x^2 - 7e^x + 5\cos x$ son:

$F(x) = ax^3 - be^x + c\sin x$
$f(x) = 3x^2 - 7e^x + 5\cos x$

F'(x)= f(x)

$$F'(x) = 3ax^2 - be^x + c\cos x$$

a= 1 b=7 c=5

La respuesta correcta es la b)

2) La integral indefinida $\int x\sqrt{2x^2+3}\,dx$ vale:

$$\int x\sqrt{2x^2+3}\,dx = \int x(2x^2+3)^{\frac{1}{2}}\,dx = \frac{1}{4}\int 4x(2x^2+3)^{\frac{1}{2}}\,dx =$$

$$\frac{1}{4}\cdot\frac{(2x^2+3)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{\sqrt{(2x^2+3)^3}}{6} + C$$

La respuesta correcta es la b)

3) La integral $\int \frac{sen2x}{sen^4x + cos^4x}dx$

$$hacemos\ el\ cambio\ t = sen^2x \quad dt = 2senxcosxdx = sen2xdx$$

$$cos^4x = (cos^2x)^2 = (1 - sen^2x)^2 = (1-t)^2 = 1 - 2t + t^2$$

$$\int \frac{sen2x}{sen^4x + cos^4x}dx = \int \frac{dt}{t^2+1-2t+t^2} = \int \frac{1}{2t^2+1-2t}dt = \int \frac{1}{2\left(t^2+\frac{1}{2}-t\right)}dt =$$

$$\int \frac{1}{2\left(t^2-t+\frac{1}{2}\right)}dt = \int \frac{1}{2\left(t^2-t+\frac{1}{4}+\frac{1}{4}\right)}dt = \int \frac{1}{2\left(\left(t-\frac{1}{2}\right)^2+\frac{1}{4}\right)}dt = \frac{1}{2}\int \frac{1}{\left(t-\frac{1}{2}\right)^2+\frac{1}{4}}dt =$$

$$\frac{1}{2}\cdot\frac{1}{\frac{1}{2}}\arctan\left(\frac{t-\frac{1}{2}}{\frac{1}{2}}\right) = \frac{1}{2}\cdot\frac{1}{\frac{1}{2}}\arctan\left(\frac{sen^2x-\frac{1}{2}}{\frac{1}{2}}\right) = -\mathbf{arctan(cos2}\boldsymbol{x}\mathbf{)} + \boldsymbol{C}$$

La respuesta correcta a este apartado es la d)

4) Al integrar por partes $\int \frac{x\cdot e^{\arcsen x}}{\sqrt{1-x^2}}dx$ *se obtiene*:

$$x = \operatorname{sen} u\,,\quad u = \operatorname{arcsen} x,\quad du = \frac{1}{\sqrt{1-x^2}}dx$$

$$\int e^u \sin u\ du = -e^u\cos u - \int(-e^u\cos u)du = -e^u\cos u - \left(-\int e^u\cos u\,du\right)$$
$$= -e^u\cos u - \left(-\left(e^u\sin u - \int e^u\sin u\,du\right)\right)$$

Por lo tanto $\int e^u\sin u\ du = -e^u\cos u - (e^u\sin u - \int e^u\sin u\,du)$

Despejamos $\int e^u\sin u\ du = -\frac{e^u\cos u}{2} + \frac{e^u\sin u}{2}$ *sustituimos en* $u = \textbf{arcsen x}$

$$-\frac{e^{\text{arcsen x}}\cos(\text{arcsen x})}{2}+\frac{e^{\text{arcsen x}}\sin(\text{arcsen x})}{2}\ simplificamos = -\frac{1}{2}e^{\text{arcsen x}}\left(\sqrt{1-x^2}-x\right)+C$$

La respuesta a este apartado es la d)

5) La integral $\int \frac{2x+2}{x^2+4x+13}$ vale

$$\int \frac{2x+2+4-4}{x^2+4x+13}dx = \int\left(\frac{2x+4}{x^2+4x+13}-\frac{2}{x^2+4x+13}\right)dx = \int\frac{(2x+4)}{x^2+4x+13}dx - \int\frac{2}{(x+2)^2+9}dx =$$

$$= \ln(x^2+4x+13) - \frac{2}{3}\arctan\frac{x+2}{3} + C$$

La respuesta correcta a este apartado es la a)

6) La integral $\int \frac{dx}{sen^2x cos^2x}$ vale

$t = \text{tgx}$	$\text{sen x} = \frac{t}{\sqrt{t^2+1}}$	$\text{sen}^2\text{x} = \frac{t^2}{t^2+1}$
$dx = \frac{dt}{1+t^2}$	$\cos x = \frac{1}{\sqrt{t^2+1}}$	$\cos^2 x = \frac{1}{t^2+1}$

$$\int\frac{dx}{sen^2x cos^2x} = \int\frac{\frac{dt}{1+t^2}}{\frac{t^2}{t^2+1}\frac{1}{t^2+1}} = \int\frac{t^2+1}{t^2}dt = \int\left(1+\frac{1}{t^2}\right)dt = t-\frac{1}{t}+C = tgx - cotgx + C$$

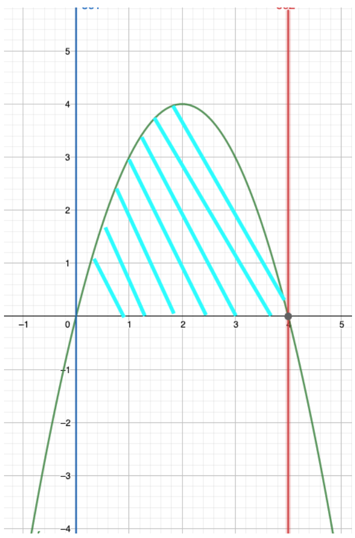
La respuesta a este apartado es la d)

7) La integral definida $\int_0^{\pi}\cos x\,dx$ vale:

$$\int_0^{\pi}\cos x\,dx = \left.\sin x\right]_0^{\pi} = \sin\pi - (\sin 0) = 0$$

La respuesta correcta es la c)

8) Para hallar el área comprendida entre la función $f(x) = -x^2+4x$, el eje de abscisas y las rectas x=0 y x=4, debemos representar dicha función y ver el área que comprende:



Una vez que tenemos la gráfica, y vemos donde corta la función con el eje y con las rectas, comenzamos a aplicar la regla de Barrow para obtener el área.

IES ATENEA Ciudad Real

Revisor: Luis Carlos Vidal del Campo

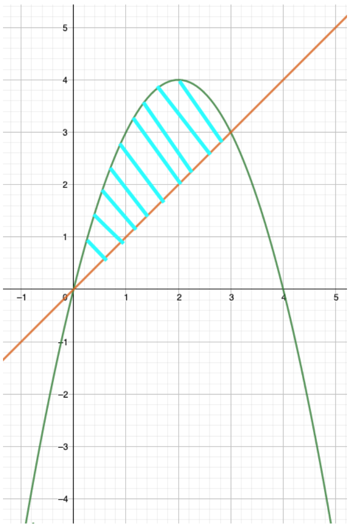
Ilustraciones: Creadas con GeoGebra

$$\int_0^4 (-x^2+4x)dx = -\frac{x^3}{3}+\frac{4x^2}{2}\Big]_0^4$$

$$\left(-\frac{4^3}{3}+\frac{4\cdot 4^2}{2}\right)-(0)=\frac{32}{3}$$

La respuesta correcta es la b)

9) Para hallar el área comprendida entre las funciones $f(x)=-x^2+4x$ y $g(x)=x$, debemos representar ambas funciones y ver el área que comprenden:



Una vez que tenemos la gráfica, y vemos el dónde corta f(x) con g(x), debemos sacar los puntos de corte y, una vez hallados comenzamos a aplicar la regla de Barrow para obtener el área.

Puntos de corte: Para hallarlos debemos igualar las funciones y despejar la incógnita "x".

$$-x^2+4x=x$$

$$0=x^2-3x$$

$$0=x(x-3)$$

$$x=0$$

$$x=3$$

Ahora ya podemos aplicar la regla de Barrow:

$$\int_0^3[(-x^2+4x)-(x)]\,dx=\int_0^3(-x^2+3x)dx=-\frac{x^3}{3}+\frac{3x^2}{2}\Big]_0^3$$

$$\left(-\frac{3^3}{3}+\frac{3\cdot 3^2}{2}\right)-(0)=\frac{9}{2}$$

La respuesta correcta es la a)

IES ATENEA Ciudad Real

Revisor: Luis Carlos Vidal del Campo

Ilustraciones: Creadas con GeoGebra

10) El volumen del sólido de revolución generado por $y = x^2$, entre 0 y 2, al girar en torno al eje de abcisas es:

$$\mathbf{V = \pi \int_0^2 (x^2)^2 dx = \pi \left(\frac{x^5}{5}\right)\Big]_0^2 = \pi \frac{32}{5}}$$

La respuesta a este apartado es la d)

IES ATENEA Ciudad Real

Revisor: Luis Carlos Vidal del Campo

Ilustraciones: Creadas con GeoGebra